Philosophy Delineated,

Containing a

RESOLUTION

Of divers

Knotty Queſtions

Upon ſuddry

PHILOSOPHICAL NOTIONS:

V I Z.

Concerning the Original of Springs, and of their Irregular
Ebbings and Flowings.

Of Chymical Multiplications or the increaſing of Quantity of
Liquors by Diſtillation.

Of the poſſibilty of a Perſons Learning in an hour or two to
write his Mind in a Language he is ignorant of.

Concerning Sympathy between ſeveral Parts of Animal bodies,
Freſh Water Springs are Aſſerted at the Bottom of the Sea.

Of the Sutures in the Skul.

Of the Conſiſtency of Immenſe Volatility with Immenſe
Ponderouſneſs

Of the Motion of the Celeſtial Bodies about their particular
Axes.

Concerning Angles of Contaᴄt.

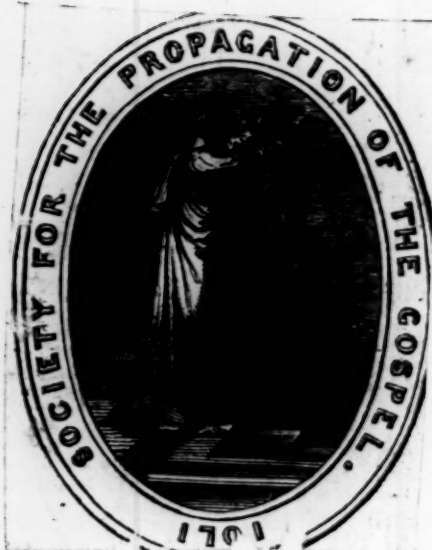With a large Cut of Braſs explaining divers

PROBLEMS.

Cum multis aliis ⸺

By *William Marſhall*, Dr. of Phyſick of the College
of Phyſicians, *London.*

London , Printed for *Obadiah Blagrave*, at the *Bear* in St. *Paul's*
Church-Yard, near the Little North Door, 1678.

SOCIETY FOR THE PROPAGATION OF THE GOSPEL. 1701

1393 ~ 26

To the right Honorable *William* , Lord
Vifcount *Bronker* Prefident , and the
reft of the learned members , of the
Royal Society.

To the Incomparably learned Dr. *Fran-*
cis *Gliſſon* Prefident , and the reſt of
the learned fellows of the, Colledge of
Phyficians in *London*.

To the worthily famous Profeſſors of
Gresham Colledge in *London*.

The Author wiſbeth all happines, ſubmitting
theſe his Philoſophical anſwers to their
grave cenſure.

THe things here ſprefented , and
which fhall be hereafter prefented
of the like nature , will vindicate
mee from both fufpition in you , and ex
pectation in the Reader , of a dedicatory
piece of flattery : I every where mani-
feftly ufing a Philofophical freedome.

A 3   And

And I am not so ill a servant either to your Honours, or to noble truths, which are the choice food of refined minds, to make this address unto you for the protection of any thing which may deserve a black censure. Let every line bear its own fate in its own bosom. Only being well assured, that these things in difference, and thus discussed between me and some friends; as they have been contrarily taught by persons worthily had in singular veneration, both in the present and former ages; so they concern some heads of Philosophy, which are of no small consequence unto true learning; and besides, lying deeper in the pit of obscurity then the first fadom, they ought to receive the ablest decision; I knew not therefore how more candidly to strip my self of any concern pervicatiously to defend any thing in these papers, then by laying them and their fate at the feet of so noble and able Judges. Possibly the thanks of pri-

vate

vate friends may have begot in me a better opinion of some of these things then they may deserve: your judgment may correct the errour of their indulgence. As all that desire to learn, ought to wait upon you and your honourable societies, as being togetherwith the famous Universities and our Law Colledges, I mean our Innes of Court, as the soul and spirits of the Nation; so these Papers wait upon you, not presuming in the least to inform, but submissely craving rather to learn of you what they ought to think of themselves: according to the noble method of Philosophy in some of your societies lately instituted and pursued, not seeking for a precarious allowance, as a Rogue and Felone may find refuge and safety at an holy sanctuary; but (however without the overweening dotage of self opinion) either by their own fair truth and clearness to be justifyed, or else willingly to embrace the flames, which their er-

A 4   rours

rours have deferved. I know you will pardon the hafty rude dreffe, or rather nakednes, (beft fuiting the purfuances of truth) in which thefe Anfwers to the queftioning letters of fome private friends, were firft of all conceived: which without falfiifyng could not afterwards be altered: and their higheft ambition then was only private fatisfaction, though now, except I will leave thofe, for whofe fatisfaction they were firft written, unfatisfied, they muft proceed to put themfelves upon the teft for publick, either juftification, or difallowance. The which I am the more eafily perfwaded to, obferving it a method allow'd and practiced both by antients and moderns to preferve and deliver fcattered points of Philofophy by way of Dialogues and Epiftles. And indeed this method hath this fpecial advantage in it; we may in a word prefent what we have to fay that is new, without loading the Reader with an unfavoury

cramb,

crambe , which hath been a thousand times before in the press. As if these paters be able, which I leave to your serious censure, to justify themselves: it is scarce possible for them to fall into the hands of any Reader, who shall not herein, if he read them diligently and with understanding , find, not only somewhat which is new , but as well that is memorable , and beyond all doubting , certain, and perhaps neither by himself formerly thought of , nor possibly every where in that vast Ocean of Books, which are in the world, to be met with. In these philosophical oppositions, till I be called to account , I have for the honour I bear them , spared those great and venerable names both moderns and antients, which I have herein opposed: it being my method in private, to pursue what is publickly in you exemplary *Viz.* to weigh controversies and questions in philosophy , not by parties and factions, time and conti-

nuan-

nuance, or number and opinion, but as near as poſſibly I can, by obſerving how the ſcales bear between the oppoſite reaſons : and I wiſh the method might be ſo happy, as to become more common. For the ſame reaſons I have ſuppreſſed the names of my ingenious friends, in oppoſition to ſome of whoſe thoughts theſe things were firſt of all written. Every day make your names more illuſtrious for clearneſs of judgment in the myſteries of nature. Being the deſire of all, and doubtleſs expeſtation of moſt and particularly of your Orator and ſervant

William Marſhal.

T'

To his worthy and learned friends with whom these things were first debated; the Authors apology for the publication of these his papers.

TO some of you the name of an Apology might seem almost unintelligible, or little less then mockery : your interest being no less in the birth, then in the first conception of these philosophical discourses. Your quærying curiosity at first imprægnated the mind, with what your present importunity will not rest without committing to the Press. To the rest of you, my wish is, that with the favour of others, I might not have stood in need of this Apology. However let not the publication be mis-construed, as if I accounted all here debated to be peremptorily concluded, or all that I have opposed, conquered. The yet dissenting of several of you from several of the things here by me own'd, might make a more hardy confidence then, I hope, I shall ever be guilty of, to become modestly jealous

of

of its own judgment. If the things here made publick fhould not be able to indure the piercing aire of this knowing age, however our free and mutual intercourfe upon fuch and the like philofophical doubts cannot want its juftification. The publifhing fometimes of a miftake is an occafion of leading others into the right way. And becaufe every one cannot fee crofs the Mediterranean, we are not therefore to walk either with hooded eyes, or to dif-own the feeing of any thing at all, or what appearances things make unto us. Though the mind, which is the fouls bed, be neither in all of the fame fize, nor in all equally luminous, or conceptive, yet each is equally concerned diligently according, to the proportion of his window, to take a view of the goodly fcenes of nature. In thefe things 't is better to erre then to be idle: as I am fure you are equally perfwaded with

Your faithfull friend and fervant
WILLIAM MARSHAL.

TO THE
READER.

THe publishing these papers hath now made thee my judge and censor: and they were no otherwise designed, but for every one freely to exercise his judgment about them. Most of these Problemes are more frequently upon the Anvile then they are found easily malleable into fair, clear, doubtless conclusions. Therefore if we misse the goale, it is but allowing one more in these sublime things to run a planetary course. I shall in a word tell the worst, which can justly be laid to their charge; some of the few things here discoursed, have left the commonly troden path; not as affecting novelty, against which upon other occasions I appear for the antients

tients, but as chusing to follow reason rather then the name of any Authour. First therefore examine their reasons, and then give unbiassed judgment for the truth; which for the truths sake I shall be ready to accept, though against me. For if those to whom and against whom, though in a vayled way, these papers were first written, are supposed not to be exempted from errour, there is no room left but only for weak minds in these secrets of nature to arrogate unto themselves an infallibility. If any complain of obscurity, as both hast, and brevity, and the subject matter, and the manner of writing, and the quality of the persons for whom these things were first of all pen'd, may make them thereunto lyable enough, however I shall be ready to give both my account, and if valuable, my assistance to any who shall desire it relating either to the obscurity, or in-

tri-

tricacy of any paſſage ; and to ac-
cept from them any reaſons they ſhall
be pleaſed to produce to the con-
trary,

                              Thine
                              W. M.

From my Study in
Nags head Court
in Grays Church
Street.

The

A SYLLABE

of the

CONTENTS

of these two

SECTIONS:

In the first Section.

In the first Answer; is treated

Far

In the Third Answer.

That,

That , *in the Doctrine of the Ancients , seemeth a greater number of first Elements asserted , then four.*

What may have been the true rational intendment of the Ancients , in their introducing the Doctrine of the four Elements.

In the Fourth Answer ; is treated

Of the Sutures in the skull : and their distinctness according to their several kinds , number , and order.

How in the distinction of the sutures , both as to their number and order , is a certain latitude , admitting variety in the method of their account , and the Anatomical Doctrine of them.

The like latitude is observable in several parts both of Concrete , and Abstract Mathematicks.

Whether the præeminence of right-side parts above the left , be natural or arbitrary.

In the Fifth answer ; is treated

Of the consistency of immense volatility with immense ponderousnes.

Of making tinctures by materation without addition.

Whether predictions may be concerning Comets before their appearance ?

What is herein to be judged, of Comets which are by new amassement ?

What is herein to be judged , of Comets which are

sup.

*suppoſed to be permanent Bodyes , and not to be
new amaſſements.*

In the Sixth Anſwer ; is treated

*of the motion of the Celeſtial Bodyes about their
particular Axes.*

*Whether ſuch their Circumaxall motion afford a
convictive demonſtration and proof of the Co-
pernican Syſteme?*

*That the Earths motion is not ſufficiently proved ,
upon the Hypotheſis granted , That Comets are
neareſt the Earth , when they are in oppoſition
unto the Sun.*

In

In the Second

SECTION.

In the Seventh Anſwer, is treated

COncerning *Angles of Contact.*
*In order thereunto , are given ſeveral deſi-
nitions of Plane Angles , and diſtinct names
to them , according to thoſe diverſityes of their
kinds and conſtitutions , which , in this Que-
ſtion , are more eſpecially material and neceſ-
ſary to be diſtinguiſhed.*

The ſtate of the Queſtion is ſet forth.

*It is ſhewn, that Recto-convexe Angles of Contact
are truly Angles according to the definition of
Plane Angles.*

*That Recto-Convexe Angles of Contact are neither
deſtitute of Quantity , nor their ſides coinci-
dent.*

*In what ſeveral wayes , one Quantity may be grea-
ter then another.*

Of Homogeneity.

*What kind of Homogeneity it is , that is requiſite
for proportionals ?*

*That Angles have Figuration , as well as Quan-
tity.*

That in Angles , is obſervable an Homogeneity or
                                         He-

Heterogeneity which is Mathematical, and also an Homogeneity or Heterogeneity which is not Mathematical, nor any way concerning their Quantity *, but only their figuration ,* &c.

How three Angles being all Equal ; the Equality between the first and second , may be more absolute then the Equality between the first and third being only respective.

How some Angles are constituted by the Concrescency , Composition , and Annexion of several parts and Angles , which are Heterogeneal each to other.

VVhether like Angles , in unequal Circles and in like segments of Circles be alwayes equal ;

That all Equal Angles are not like.

That all like Angles are not equal.

VVhether this Controversy about the Recto-Convexe *Angle of Contact, belong to Concrete or Abstract Mathematicks?*

VVhat was the first mistake , that first ministred occasion for the starting of this controversy , which hath been so long reteined , in these Learnings , in which nothing is more monstrous and unusual then pertinacious controversy ;

These Escapes of the Press crave the help of the Readers Pen. *viz,*

PAg. 10. line. 11. read. aerial. p. 17. l. 2. r. heat. l. 16. 17. r. reason. p. 18. l. 10. r. easily. p. 14. l. 19. expunge the semicolon. p. 21. l. 28. r. Sympathy. p. 22. l. 15. 16. r. tumultuous. l. 24. r. those. p. 23. l. 29. r. concerning. p. 24. l. 18. r. falls. p. 29. l. 7. r. confessedly. p. 33. l. penult. r. flushing. p. 35. l. 13. r. that that. l. 28. r. fire. p. 36. l. 13. r. us. p. 39. l. 1. expunge the colon : l. 16. r. fætid. l. 26. r. their. l. 27 r. the. p. 41. l. 7. r. with the string. p. 43. l. 12. r. to the violating. p. 44. l. 2. and 7. r. inanimates. p. 49. l. 3. r. seame p. 56. l. 9. r. certainly. p. 57. l. 15. for is regularly. r. irregularly. p. 61. l. 7. for if seeming. r. it seeming. l. 19. r. exact. p. 69. l. 5. r. limited. p. 71. l. 3. r. the Copernican. p. 75. l. 13. r. Jago. p. 80. l. 19. r. Lordships. p. 82. l. 17. r. like. p. 86. l. 5. r. BA. p. 89. l. 16. for mooving by. r. by mooving. p. 90. l. 2. r. an isoclitical. l. 28. r. Be again. p. 92. l. 30. for AHH. r. AHF. p. 100. l. 6. r. even. l. 19. r. say. p. 101. l. 1. r. one or each. p. 124. l. 29. r. recto-concave. p. 155. l. 26. and 28. r. crooked-lined. p. 160. l. 18. r. not other. p. 187. l. 26. r. a mixt-lined secant angle or of p. 200. l. 26. r. crooked-lined. p. 232. l. 1. r. constant.

The faults of the Orthography are referred to the Readers ingenuity.
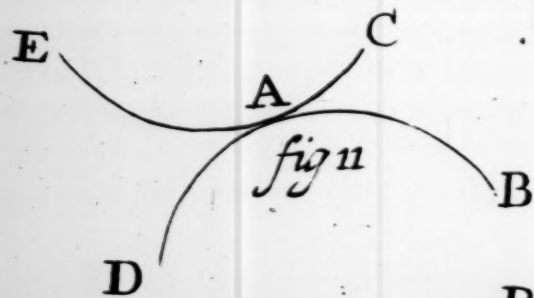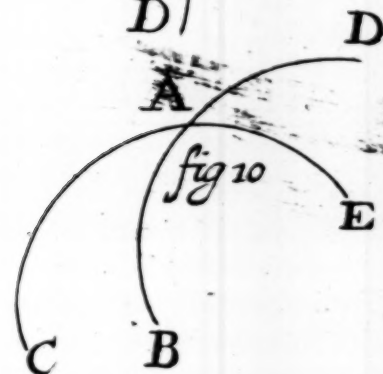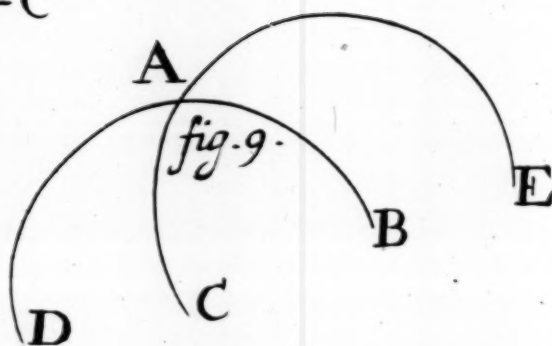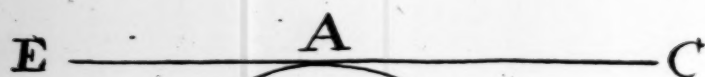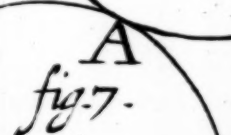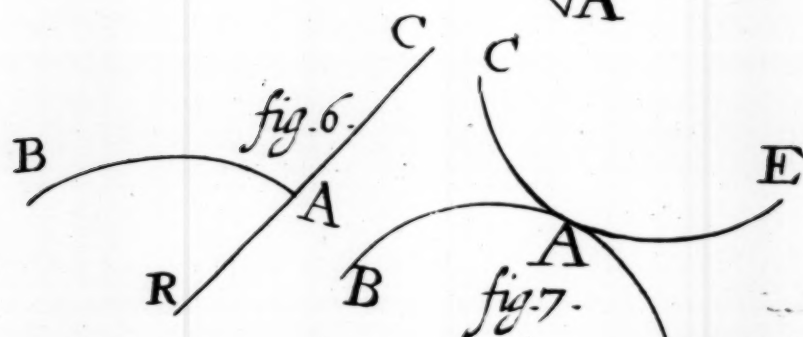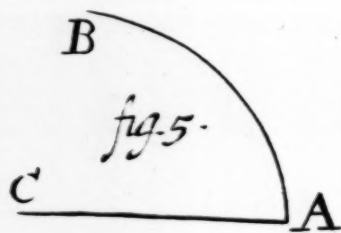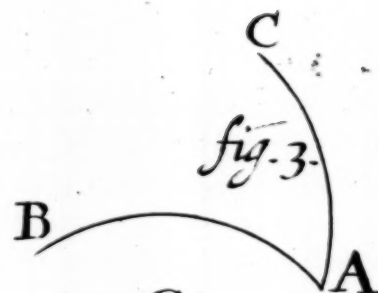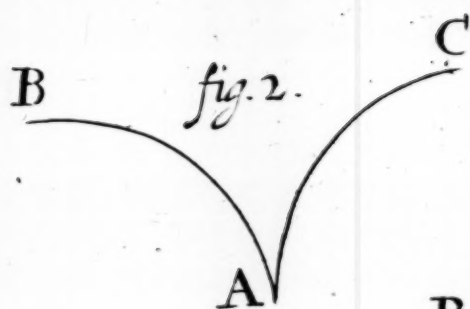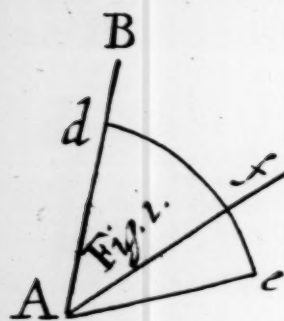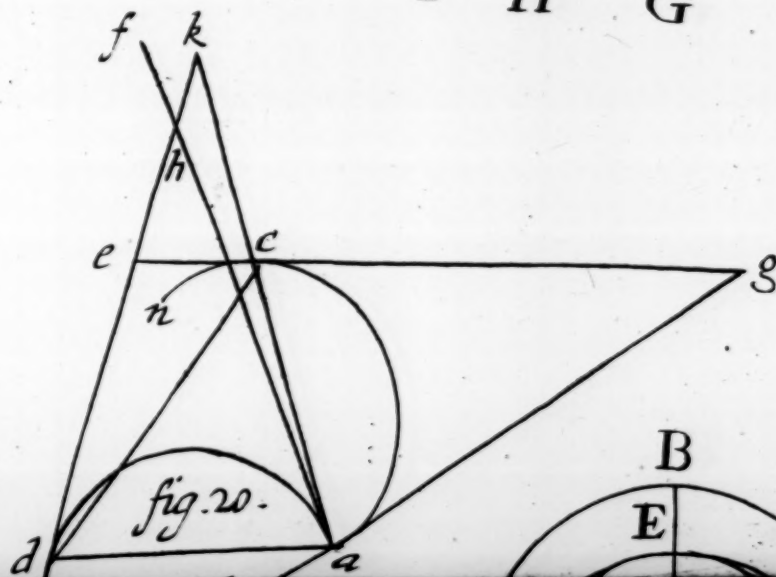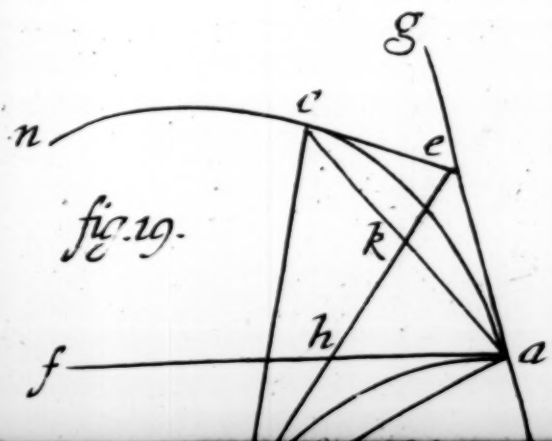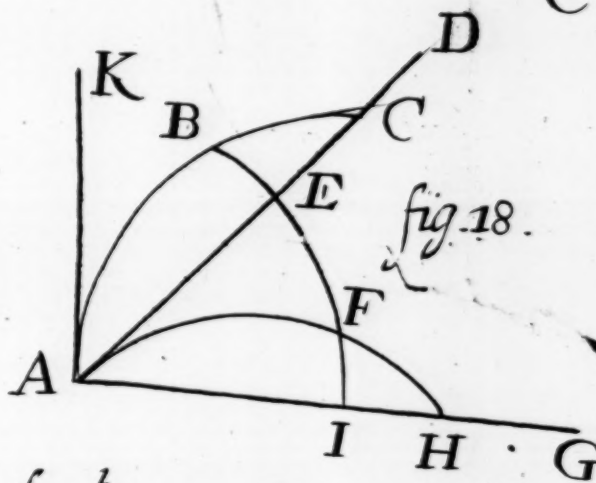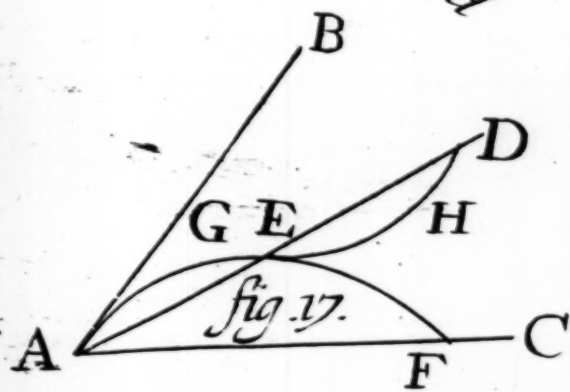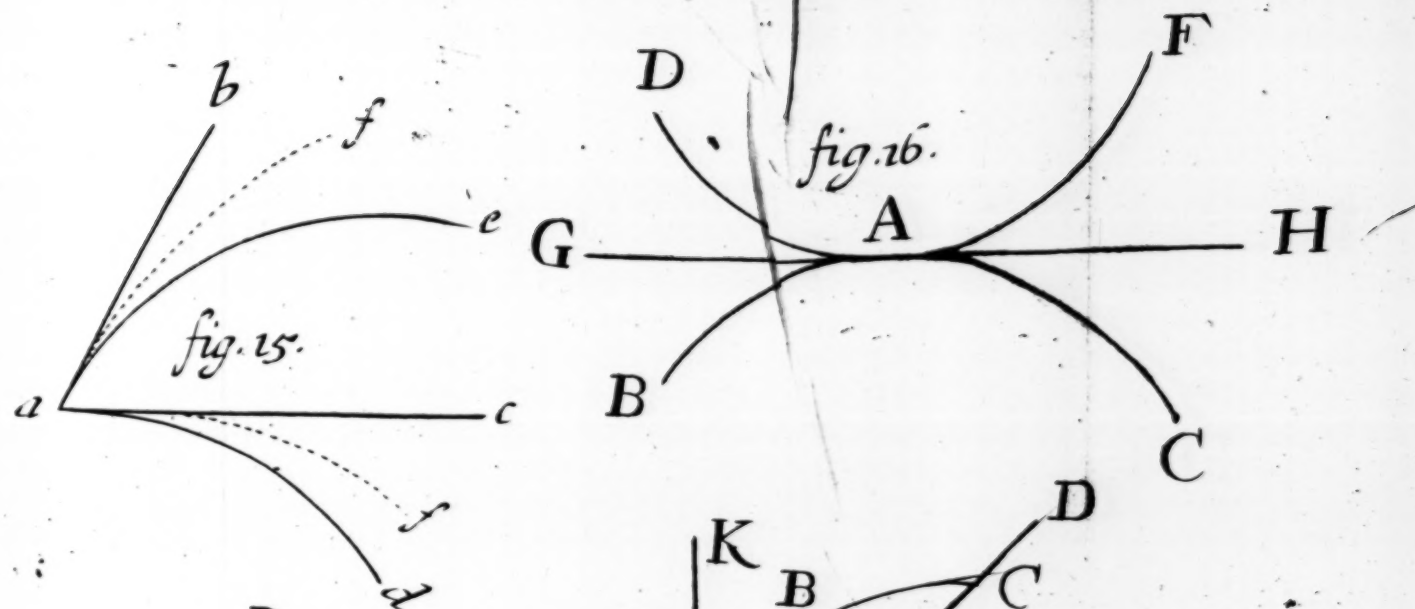
B

d

C

f

Fig. 1.

A

e

B

fig. 2.

C

A

C

fig. 3.

A

B

A

fig. 4.

C

B

B

fig. 5.

C

A

B

fig. 6.

C

A

R

C

E

B

A

fig. 7.

D

D

A

fig 10

E

E

A

C

fig. 8.

D

B

A

fig. 9.

E

D

C

B

C

B

E

C

A

fig 11

B

D

A

L

B

E

fig. 13.

H

M

D

C

G

N

F

S

R

H

K

b

f

e

G   A   H

fig. 15.

a

c

D

F

fig. 16.

B

C

f

d

B

D

K   B   C

G   E   H

E

fig. 18.

A   C

F

fig. 17.

A   F

I   H   G

g

c

f   k

e

c

n

h

e

c

n

g

fig. 19.

k

f   h   a

fig. 20.

B

E

f   d   a

E ———————— A ———————— C

fig.8:

D

B

E

C

A

fig.11

B

D

A

fig.9.

B

D

C

E

D

A

fig.10

E

C

B

S R

B

H

fig.12.

K

G

A

D

F

L

E

B

C

A

L

B

D

C

E fig.13. H

G

K

F

N

M

F

E

B

A

fig.14.

D

H

G

C

G E   H
A   D
fig. 17.
F   C

A   F
I   H   G

fig. 19.
n   c
e
f   h   a
k
d
b
g
l

f   k
h
e   c   g
n
d   fig. 20.
l
a
b

B
E
C H   A   fig. 21.   G D
F
K

g   a
c
fig. 23
e   f
d   g
n   a
b

g   a
c
fig. 22
e   h   b
f   d

fig. 24 ————— A
————— B
D ————— C
E   F
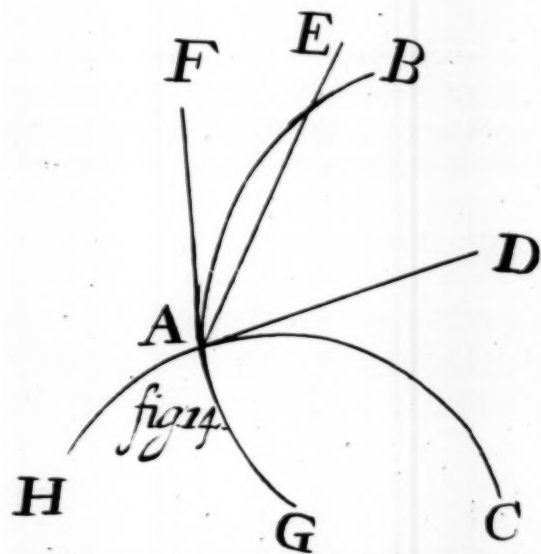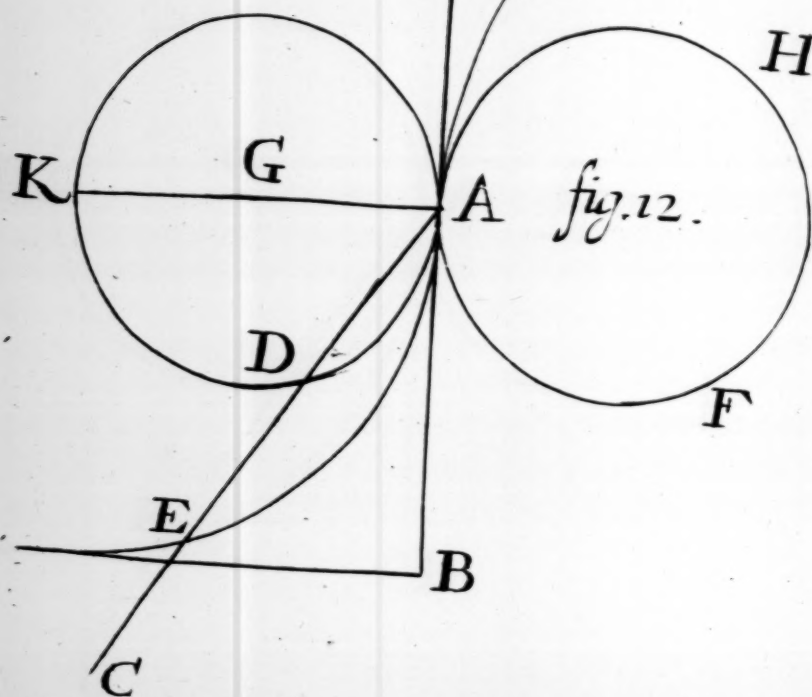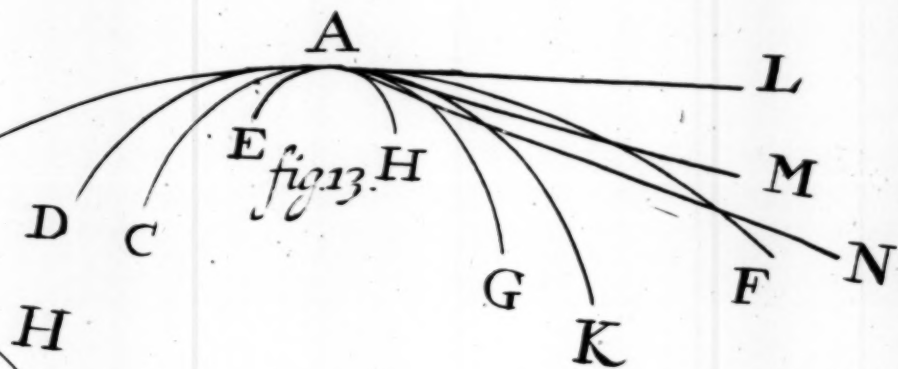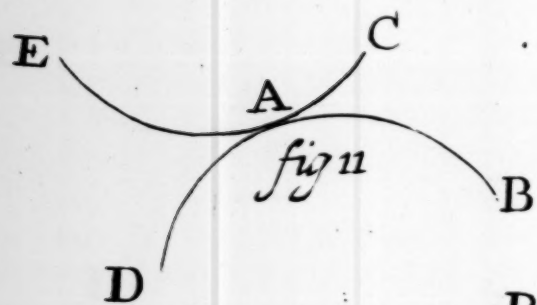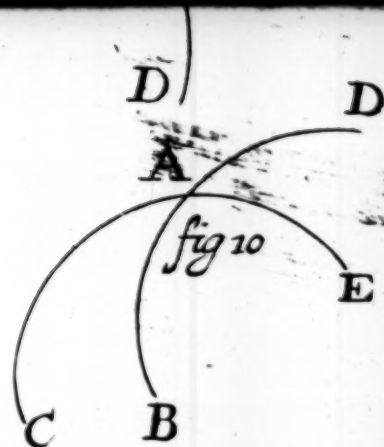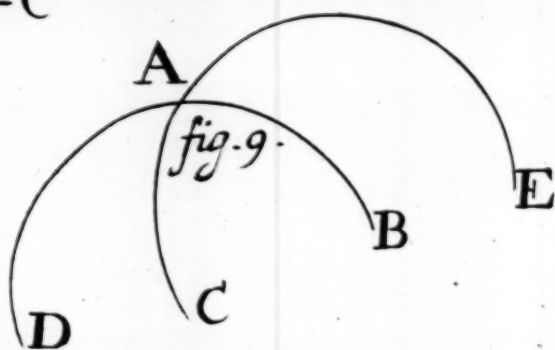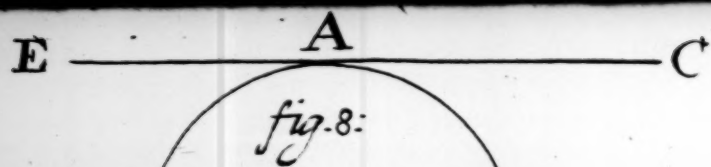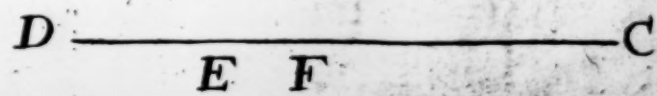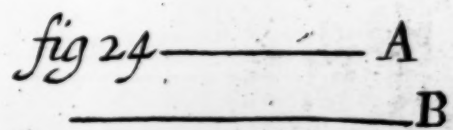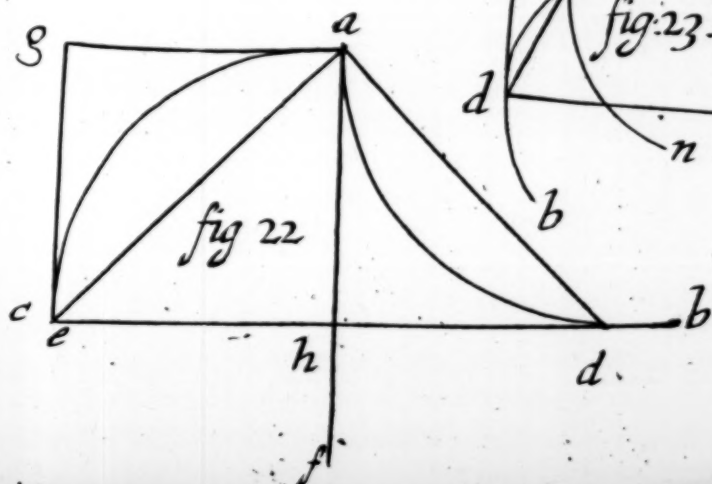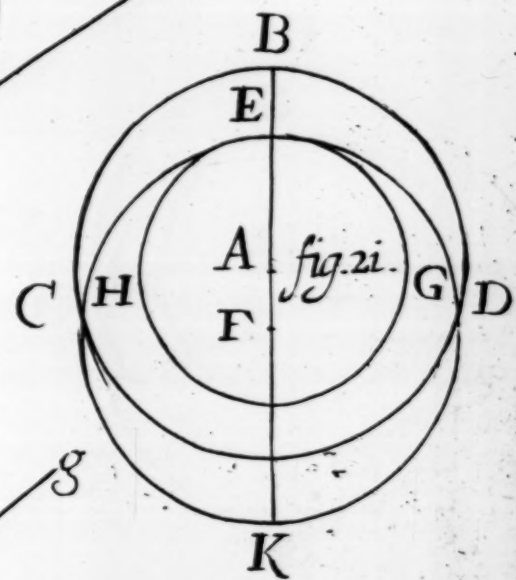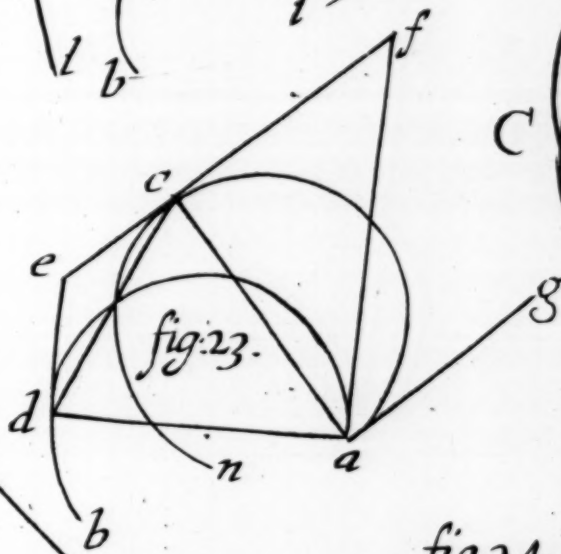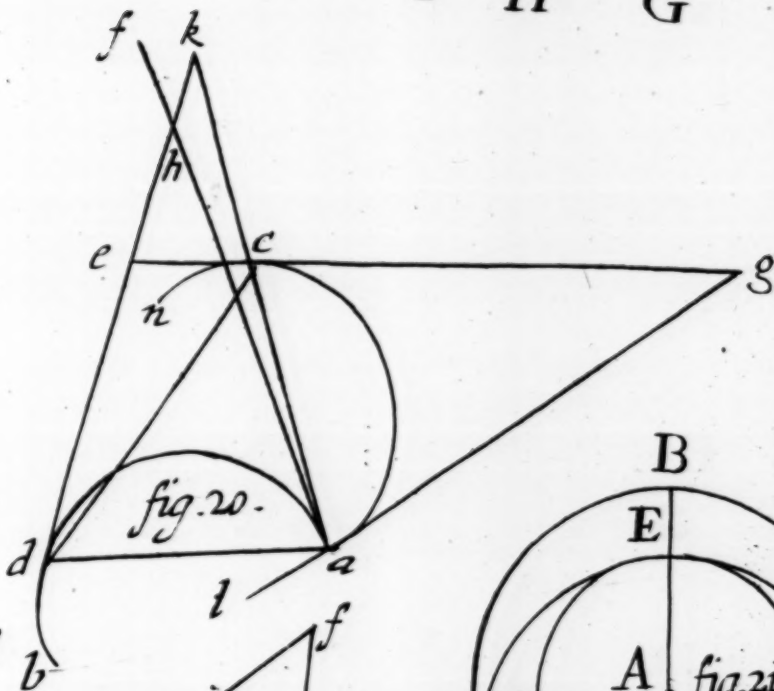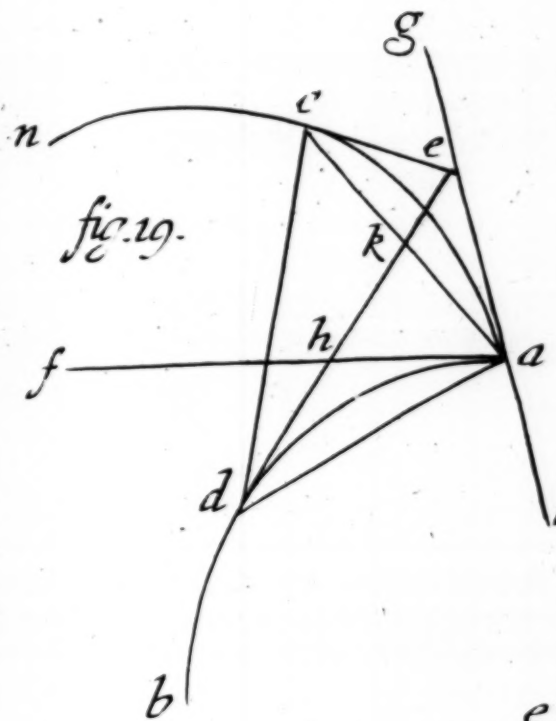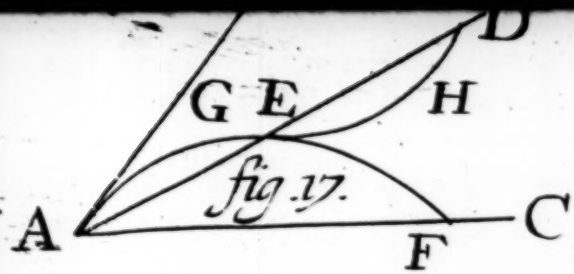
ANSWERS

Upon several heads in

PHILOSOPHY

The first Answer.

Concerning the original of Springs: that all Springs have not the same nor the like original: that all Springs are not from the Sea: that several things may by the way contribute as originals to the same Spring, whether by way of dissolution, or condensation. Also of the ebbings and flowings of particular Springs: in some more especially observed to be in an unaccountable irregularity. Also of Chymical multiplication, or the distilling of liquors, so as by distillation to encrease their quantity to any given proportion. That Mercury is not explicable by the Doctrine of the four Elements. That in an hour or two's time a person of indifferent parts, able to read and write his native language only, may be taught to write his mind in a forein language, so as what he writeth shall be true in the forein language, and the sense good and continued.

AS Nature in generation worketh the first beginnings of things usually in a very dark loom, not permitting common eyes to have a view of its untaught art in ordering and warping its first filaments, covering with shells and membranes and divers veils the mysterious na-

ture

ture of firſt productions : The ſame judgement
may be of Springs , which though every where
obvious to the eye , by their pleaſant murmu-
rings , cryſtalline pureneſs, and perpetual flux ,
delighting the mind , as well as the ſenſe ; yet
whence theſe Riſe , and how they are fed, and
wherewith furniſhed , is more obſcure and æ-
nigmatical than to be reſolved , as many eaſily
imagine, in a word. And yet I ſuppoſe , an er-
roneous *Hypotheſis* may be the chief ground of
difficulty in this Queſtion , while many gene-
rally expect that all Springs ſhould be of the
ſame or like original. But why is that more
reaſonable than to expect that all Springs ſhould
ſend forth waters of the ſame taſte , colour, or
virtue? Too many Inſtances , Examples and
Experiments may be produced of Springs ow-
ing their original to the Sea , percolated tho-
row the Earth , to deny, or call in queſtion ſo
evident a Truth, without a manifeſt crazing of
our own judgement : yet that the Sea by ſuch
percolations is not the original of all Springs, is
as manifeſt, as true, and confirm'd by as many
and weighty Experiences ; not only of freſh
Springs near the Sea, and in-land ſalt-ſprings ;
but eſpecially if we conſider the numerous
Springs , which are in the tops, or ſides of ſeve-
ral Mountains, vaſtly above the Seas level; and
therefore not poſſible in any Channel to be na-
turally raiſed ſo high , without all ſtay, or im-
pediment

pediment of percolation. Of which I have ob-
ferved a fpecial inftance in a low, fomewhat
rocky, wedge-like Mountain, fituate along
the brim of the full Sea; but on the Land-fide,
at a little diftance, fet about with numerous and
vaft Mountains: in which little wedge-like
Mountain, though moft part immediately en-
compaffed with a Plain, very confiderably a-
bove the level of the higheft, æquinoctial, or o-
ther Tides; was a perpetual Spring of moft
lympid, clear water, not unfamed for its fingu-
lar medical vertues. It is fcarce confentaneous
to fenfe, reafon and experience, to derive fuch
Fountains out of the Sea only by way of perco-
lation; which though it be allowed to contri-
bute, in fome places, to the fweetning of the
fountainous flux, cannot be conceived apt to
raife the water to an higher level than it had
before. Not difallowing the former therefore
in its place; there is yet befides it fome other
Original of Springs to be inquired after. And
how perpetual Springs fhould come to be in the
tops of the higheft Mountains of all, as the cafe
of moft difficult explication, cannot with great-
er reafon and clearnefs be explain'd, than by
bringing the waters thither, not in a watry
form, but as vapours and exhalations, at leaft-
wife the moft conftantly, and in the greateft
proportion; after the manner of a Diftillation,
ordered and managed by the inftitutes of Na-
ture;

ture ; and this without Espousing the Tenets
of our new Platonists ; that will have several
perpetuous continued Orbes of Fires , diversly
graduated , for special Theological ends , con-
tained still between two and two perpetuous
and continued Crusts, or sphæral shels of Earth,
the one above the fire , the other beneath it ,
in respect of the Earths Center. And those that
know how under the surface of the Earth ,
whether plain , or mountainous , are frequent
stores , both of Materials , and naturally form-
ed Cavities, and other Instruments , easily ac-
commodable to such a work , will not conclude
that to be a conjecture of fancy , which not on-
ly for its possibility , but the high degree of its
probability , may justly claim to be entertain'd
as a most real and undoubted physical Truth.
For what doubt can be made of subterraneous
heats and fires ? and hot Baths and Springs,
attest the, not only warmth , which is sufficient
for our purpose , but even the actual , and fre-
quently intense, heat, and ebullition of liquors
within the Caverns of the Earth , and the per-
petuity, or constant continuance of such ebulli-
tions : and as the Earth is well known to be
furnished with many natural Caverns , so in
those Caverns, for shape, proportion, situation ,
and other the like circumstances , we cannot
but imagine there is great diversity. So how
variously and plenteously the Earth is watered
with

with fubterraneous juyces, and moistures, cannot
be denied by the obfervant, who almoſt every
where under ground find leſſer veins of water
difperfed up and down the Body of the Earth,
and in many places large concealed Rivers,
elſewhere unfathomable Vaults, and Abyſſes:
Sometime feen Rivers in their proper and per-
petual Channels, make a dark courfe and dif-
charge of their waters into the unfeen Bowels
of the Earth. Upon the whole, in the method
of this explication, waters are, in the fame
manner fecretly within the Earth, raifed to the
top of the Mountains; in which, to our view,
they are raifed in the open Air to thofe regions,
from whence they fall back again upon us in the
form of rain. And in fuch vapid exhalations,
it cannot be denied but in fome places, and at
fome times, the Mountain fprings ranging in
a much higher level, may by pofsibility be fur-
nifhed from the percolated fea-water: but as
the poffibility of this is admitted, fo the uni-
verfal and abfolute neceffity of it, may not be
averred: it feeming hard to fay, that the Moun-
tain-fprings what diftance foever from the Sea,
have no other way of being furnifhed, but on-
ly from thence. And, I fuppofe, it will as un-
eafily obtain credit, that the Sea fhould run as
faft every way, under the Earth, to the Origi-
nals of Springs, as the Springs generally do in
feeking out their way unto the Sea. I doubt not,

but

but besides the Seas, there are under the earth many other liquors, contributing to the origination of Springs, some having their first rise from dissolution, others from condensation, whence, and from the passages thorough which they run, and are percolated, rise very often those special vertues and dangers of some peculiar Springs: and sometimes in a seeming præposterous way, though very consentaneous to the true nature of things, Springs adjoyning to the Sea are fresh, and at a vast distance from theSea, in in-land Countreys sometimes salt. But besides both the Seas,& those other waters bred under the earth; its more than probable, the rains falling down in showers from Heaven, adde not a little to the flushing and continuance of the Springs. So medical Springs after rains are noted for a while to be of less virtue ; and in long droughts, 'tis usual for very many Springs to be quite dried up, till rains fall again: and where, by long obscure dens, caverns and passages under ground, access may be had to subterraneous Rivers ; such Rivers, though lying vastly deep under the Earths surface, they are found to swell at after rains, and by their accession unto the wonted stream, frequently do make a most hideous and horrid noise, full of terrour to those in the Caves mouth, and at other times unusual. From all which, seriously and impartially weighed, is made unde-
niably

niably apparent, that rains, in no contempti-
ble proportion furnish forth matter to be ordi-
narily by channelly veins convey'd, or elfe ex-
traordinarily by a natural diftillation wrought
up into the Springs. And as this much difcuf-
fed *Queftion* is refolved in thefe eafie things,
every where offering themfelves to view in Na-
ture ; fo I doubt not, upon the fame principles
might alfo many other things, feeming at firft
fight to be very myfterious, in the concern of
Springs, as that which with fo much admirati-
on is by the rude ignorant people cryed up and
obferved in fome, having, as they term it,
their Ebbings and Flowings in fuch irregular
frequency as by no art can poffibly be reduced
to any certainty of account, or order : for the
more fpecial and clear obferving of which they
are went to receive the water from the Spring
into fome ftone, or fuch like veffel proportio-
nately bored in, or near the bottom : all which
in truth is without any retrogradation and reci-
procation of motion in its channelly veins ; be-
ing nothing elfe but an inæquality of the waters
iffuing from the Spring in equal times, either
by reafon of its more plenteous filling at fome
times the channelly veins of theSpring as it paf-
feth , or elfe by reafon it paffeth with a greater
or leffer impetus ; or poffibly in fome cafes and
places upon both grounds : which what it hath
in it that is more admirable than the running of
<div align="right">any</div>

any ordinary Brook, sometimes with a fuller channel, sometimes with a stronger stream, I do not yet understand: and how easily, upon the former principles, this is explicable any one may readily perceive : that we may more justly wonder how some grave Authors came to be so transported with the sight or fame of some such springs, occurring up and down in the World, as in their reports to offer them to the thoughts of distant Students, and succeeding times, as containing in them little less than miracle. And if the studious would observe diligently what is in nature, possibly many other things, seemingly as intricate, might be capable of as easie explications ; especially considering the vast way a Spring may run under the Earth before it break forth ; fairly insinuated to us by the pure fresh water Springs, bursting up a great way within the floud marks of the Sea ; clearly intimating to us, how two Hills at a considerable distance may both be concerned in the same Spring and its Course. And whereas we number and place Springs according as we observe them to break forth and shew themselves unto the day ; in the true æstimate, and upon laborious search, it hath many times appeared quite otherwise ; that the original of the Spring, has been at a great distance from the eruption, and the eruption has not been till after the confluxion and meeting

of, it may be three or four several Springs, eve-
ry one carrying in it sometimes the dissolution,
sometimes the spirit, or somewhat of the first
rudiments of some special Mineral; and alto-
gether make up, not seldome, an almost inimi-
table composition. Your other Quæry con-
cerning Chymical Multiplication, or the di-
stilling of Water from a Pint to a Quart, if my
Genius fail me not, when rightly proposed and
understood, cannot want some affinity with
the matters we have already been treating of.
For if any would impose that the same Water
as by way of expansion, without other additio-
nal water or matter, might by distillation be
brought to a double, treble, and so a thousand
times as large dimensions as before, as Wines,
Vinegars, and other Liquors, have in them,
without any further addition, so much Phlegme,
and so much Spirit of such a strength, whether
fixed, or volatile; it would require a very cre-
dulous judgement to allow the veracity, or
possibility of such a Probleme; and I am clear-
ly free, to disown the skill or power of any
such Chymistry; which once admitted, were
it not for the shortness of humane life, and the
hopelesness of obtaining Vessels, and Instru-
ments large enough for the work, and conve-
niency to place them, the world might be in
danger of Drowning from the lofty Artist; as
once it was hypothetically threatned, with
shaking

shaking by the noble and ingenious Mechanist. But I had rather understand a Probleme favourably, so as it may carry truth, possibility, and reason along with it : and so methinks may this be interpretable, to wit, subtilly to contrive, find out, and order expedients, ingenious helps, and advantages, to coudense free, and open exhalations into a watery form with dispatch and expeditely, especially at any season, or time of the year : so as by this Art out of the arrial vapours within a limited time to give any limited measure of water, which reason can expect from Art, or without abjuring and putting off its own nature, handsomely make demand of.   And that this is the Philosophy of Nature appears in every days dewes, the trickling moisture hanging on the chamber-side of the Glass , when the Air is very sharp, and full of a nitrous coldness without: and we cannot rightly conceive the Idea of that famously known and frequently mentioned and in hot Countries, though rare, yet sometimes by sad experience attested disease , the Diabetes, without admitting both the possibility of such a Chymistry, and of Expedients for the more ready dispatch of it: for without an aptness to imbibe, attract and condense the vapid exhalations, which successively come to be near and about the Patient, and that with very singular advantages for expedition, how possibly

should

should the Patients Urine come in a few weeks to weigh more than all, both the drinks and meats taken by the Patient all along that time, and all the weight of the Body, whensoever it was heaviest during that time besides? To contract all into few, the possibility of the Probleme, according to this explication, needs not be doubted, being an every where obvious work of Nature; as in the other sense it seems vain and frivolous. And in researches about expedients for dispatch; it is possible, here, as in other pieces of that ingenious Art, several Artists may have several methods and contrivances, to suck in still more vapours in a continued succession, and proportionably to condense them when sucked in; also to dispose the vapours, before they are sucked in, for more expedite and easie condensation at after: as every one has his peculiar Still, his peculiar Solvent, Bath, Ferment and Furnace. And that this point of Philosophy were a little more cultivated were to be wished, for the advantage of natural Philosophy, and the concernes of it in some practical Arts: for so hereby the difference of the Airs at several seasons, or at the same season in several places, might come in some measure to be known, by the Taste and Eye, as well as by the Nostril: and the concerns of Rokes, Fogs, Mists, and Airs any wayes altered, or infected, may be hereby

more

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                                        more

more happily difcovered in relation to the health and ficklinefs of Man, and fo of other Animals: fo what difference there is between the Breathings of feveral Animals of the fame , or feveral Sexes and Kinds, or of the fame Animal at feveral times of its age , reft , motion, fear, joy, hope, love, anger, health, ficknefs, peculiar conditions, or evacuations: and the difcriminations between fimple Airs, and the Airs altered by fprinklings, ftrewings, fumigations, from Minerals, Vegetables, Animals, may be made more evident. Your tying up *Mercury* to the old Elemental Laws, is a true femblance of the old Mezentian cruelty ; and I fhould pity the poor Fugitive, were I not afcertained from his conftant courfe, that as a flippery *Hocas*, he will never leave tranfhaping himfelf till, to your own confefsion, he has flipt the Collar. Four qualities will affoon folve all queftions, as four Elements ; which fome of the learned Antients , vainly hoped, not to fay, foolifhly boafted, to do. As Philofophers diftinguifh between the fights of Age and Time ; Age firft fees without, and after not without Spectacles : Time firft fees with them, and after better without them : the Elements were as Spectacles in the Beginnings of thofe times, when men firft fet themfelves about the ftudies of Arts, that helped them in a fort to conceive fomewhat of Bodies ; but now

the

the Eye of time grown clearer, feeth much
more happily, as is by many suppofed, with-
out them. More crafs and thin, folid and
fluid parts are not denyed, but apparently to
be feen, offering themfelves to view in feve-
ral plain and eafie methods of analyfing Bo-
dies; but the fimplicity of thofe parts, ob-
tained in fuch Analyfmes, is not yet demon-
ftrated. The ftrange Monfter reported with
fo much averment to you, which feems fo much
to have ftartled your patience, to wit, that a
perfon of parts, educated only to read and
write the Language of the Nation wherein he
was born, may in an hours time be taught to
fet down any matter, fecret, or other, in a-
nother language, as Latine, Greek, French,
&c. and the Latine, or other Language to be
true, and the fenfe good and continued; be-
lieve me, though it be true, as it is, yet it is
not to be efteemed worthy your leaft difcom-
pofure: it may be done many feveral wayes,
and yet when all is done, it is but a flight
of the Brain; like many of the mimical Mi-
racles, performed by nimble handed *Egypti-*
ans. That there may be no room for diftruft
of the truth of it, if affiance in me may fo
far prevail; you may be affured, I have feen
the whole Model and Syfteme of it, in feve-
ral Languages, befides our own: and but that
it would make my Letter of too great a Bulk,

shoul̃d

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ral Languages, befides our own: and but that
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                                      fhould

ſhould herewith have ſent you a Tranſcript
of one of them. In this, as in the foregoing
Probleme of multiplying Chymiſtry; in words
ſpeaking and containing truth, ſomething
ſeems to be offered to the underſtanding of
the hearer, which is an *Herculian* ſhot be-
yond it. Pardon the length in which I have
laboured diſcurſively to ſend back all the
Quæries and Objections of your laſt fully
anſwered.

THE

The Second Anſwer.

Wherein of Sympathies between ſeveral parts of the ſame Animal Body in general and particularly of the Sympathy which is between the Breaſts and the Womb : and that the ground of that Sympathy is not founded upon the inoſculations of the Epigaſtrick and Mammary Veins and Arteries. That that Sympathy inſtructs not from the ſight of the Breaſt concerning the Sex of the unborn Conception. Yet parts of the ſame ſide more eſpecially Sympathize one with another, than thoſe that are on contrary ſides. That to found the Sympathy between the Breaſts and Womb, is no neceſſity of veſſels paſſing directly between the one part and the other : but the grounds of their Sympathy may be ſeveral other wayes clearly explicable, by manifeſt Arteries, Sinews, and other Veſſels. That the ſeveral ſhapes of the Moon have their original, forming, and alteration, from the parts of its mutuatitious light, and their ſituation in reſpect of our Eye. That there is not one general original of all light. And why the enlightned part of the Moon is ſtill toward the Sun. And why the Moon appears plane and not ſphæral.

I ſhould wrong the truth, if I ſhould not acknowledge, all your Arguments and Inſtances prove fairly the Sympathies, which nature has lodged in us, between ſome ſpecial parts above others ; to which may be added by way of Declaration ; that though no part in the whole animal Body, but is of affinity and con-

concern unto all the reft, yet fome, befides
their general relation as of the fame animal Bo-
dy, have fometimes not only one, but feveral
other more fpecial grounds of Sympathy, to tye
& link them one to another; whether from ho-
mogeneity of fubftance, or commonnefs of origi-
nal, or unity and concurrence in the fame
work, or concern in the fame branchings of
veffels, whether Arteries, Sinews, or Veins,
of what kind foever ; each of which heightens
the general relation which is between the parts:
and the more of thefe are found inter-curring
between any two parts, their relation is fo much
the more fortifyed. But of all other, the Sympa-
thy which you urge, between the Breafts and
Womb, in animals is too univerfally known,
and varioufly evident to be denyed, or doubted.
How oft, among prying and fufpitious Wo-
men, has the ftate of the Womb been difco-
vered from the colour of the Nipples, aud con-
dition of the Breafts ? And the judgements of
Phyficians juftified and admired, when upon
the manifeft intercourfe between thefe two
parts, eafily transferring humours, whether
natural, or præter-natural, from the one to the
other, they have fometimes predicted long be-
fore the difeafes of the Breafts, or Womb ? So
upon expectation of the ones evacuation, the
cure of the other has been ordered, and the
event and time of recovery forefeen ? and ob-
serving

serving how they grow and leſſen together; heal and fade together, become firm & flaccid together, repleniſh and empty together, except when the one drains the other, the intercourſe between them, and their common concern is manifeſt ; however the grounds and paſſages wherein this intercourſe is founded and carried on, poſſibly may not be ſo very clear, as by many is preſumed. And though I honour you for honouring the Antients ; it is now eſpecially leſs clear then ever; that the undeniable conſent and ſympathy, which is between theſe two parts, has its foundation in the inoſculations of the epigaſtrick veins and arteries with the mammeryes : Except we will ſo farr degrade the examinations of reaſons , and trials of experience, as to make what is antient, to be therefore indiſputably authentick ; Concerning the ſympathy of the Breaſts and womb as held by the Antients)there are two things well deſerving ſerious & further conſideration: For hath not this ſympathy been rackt beyond its nature, and tentered beyond the truth, of what it is able to bear, or diſcover ; when, upon the view of the Breaſts , declarations and predictions are made, not only of conceptions, & abortions in general, with a probable prognoſtick of the time and event of the feared abortion ; but alſo from the difference of the Breaſts on each ſide , the

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<div align="right">ſexe</div>

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the difference of the Breafts on each fide , the

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fexe of the conception, or abortion is peremptorily limited; and when twins of feveral fexes have been conceived, the twin of whether fexe fhall alone be aborted is forefhewn; in that rare & unufuall cafe of aborting one twin, and compleating the due months of geftation with the reft? Not to alleadge more of thefe impertinent medical Enthufiafmes; fuch groundleffe and inconfequent deductions, as they are cafily received by the fuperftitious, who yawn after things novel and ftrange, and blindly fwallow what is offered without either præexamining by the eye of reafon, or proving them by the præmaftications of experience: fo I doubt not but to fober minds they will appear an apifh overhugging the truth, to the ruine and deftruction of it: an overchurning a ferious, fecret, true, obfervation of nature into rancknefs and falfity; and a præpofterous grafting a wild, wandering fancy upon the moft cultivated ftock of fober, true and certain obfervation. For fince men have fet their minds to learn out of natures unwritten books as well as out of the writings of Authours what hath been more ufual than to find male and female conceptions indifferently on the one or other fide, or horn of the womb? And though againft experience no Argument ought to avail; we fhall not ftifl what they feem to have to fay for themfelves

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That the left præparing vein riseth from the Emulgent only, which may be suspected, to be a weakning of its power and influxion; whereas the right præparing, or spermatick vein riseth immediately from the great discending Trunk, of which the aforesaid Emulgent is but an offset. All this is generally and in most Bodies true: and in former Ages, with the glossiness of its probability, carryed away the minds and judgments of most: but now upon more curious searches made into the Bowels, and heart of nature, the sophistry, & weaknes of the former Argument manifestly appears: the spermatick veins, if any be so pleased to continue that name, being only reducing vessels, and carrying forth from the vital fountain nothing at all to the Testicles, womb, Ovaries, or other organes of generation; as may appear by observing; the motion of the juyces contain'd in them, notifyed from the side on which the tumour and fulnes suddenly upon an intercepting ligature gathereth. And the different rise of these spermatick veins is more aptly referred to the indirect situation of the discending Trunk there; which usually at the loins inclines a little rather to the right side, making the left Emulgent accordingly, to be commonly a little longer than the right: so as for a long tender venal vessel to have had its original so obliquely remote,

and

and sloping over the fore bodies of the Rack bones, might not seem sufficiently secure; however in the more strongly coated arterial vessels, it is not withstanding otherwise; the right and left spermatical arteries both rising constantly from the descending branch of the Aorta. That Argument of the right side parts being, stronger, hotter, nimbler, &c. than those on the left side; annexeth an incohærent sequel, to a Truth which is meerly accidental: the special strength of the right side parts not being any natural priviledge, but an advantage acquired by exercise, use and education; and by the same method, as easily transferable to the other side. And if at any time Twin parts be not equally concern'd, it is because there is not equall relation to the sides: so the Ovary of that Side to which the Conception is more principally affixed, in many females is of a fleshy substance, of colour like Sanders, or somewhat brighter, the other all the while remaining without any alteration as a clutter of watery Blebs & vesicles. But more particularly in the Copulations of mankind, all such considerations of sides must of necessity be obliterated; the right side of the one Parent being applyed to the left of the other, and so on the other side, the left to the right. In all this we deny not, but right side parts have more special relation to right side parts, where
the

the parts twin, and are double : but to parts, which are fingle, and pofited in the middle, Twin parts have equall relation. So as upon the whole matter, the difeafes of each particular Breaft are more prone to be transferred to the fame refpective fide of the womb, and thofe of either fide of the womb to the refpective breaft ; of which are moft apparent footfteps in the fabrick of nature : for though in fome veffels, as Blood-veins, water-veins, chyle-veins, arteries, &c. this diftinction of fides be again loft and confounded by infertion into fome common Trunk, before they can be traced from the one part, to the other collateral part ; yet in fome other veffels without any fuch confufion in the midway, the fame collateral veffels ramify themfelves diftinctly unto the parts all along on the fame fide : as is moft manifeft in the vifceral finew, or wandering pair : but yet hereby is not opened any window to fee into the fex of unborn conceptions. And, as herein I defire to reverence antiquity, but follow reafon ; fo upon your more ferious thoughts, I doubt not but you will judge, with me, another Point upon this fubject very well worthy calling in *Que-* ftion : and that is the much noyfed ground of this fympatly between the Breafts and womb; faid, and formerly thought, to be the inofculations of the mammary veins and Arteries

with

with the Epigaſtriks. It is well known, how many lateral heterogeneal inoſculations, between Arteries and Blood-veins, have been introduced by cloſet *Anatomiſts*; while, in Bloudleſs ſpeculations, they formed nature according to their reaſonings, to ſolve appearing phænomena's; inſtead of forming their reaſonings according to what they ought firſt to have obſerved in nature: the verity and authority of all which', among Anatomiſts at ſharp, that carry their Eyes in their hands, and will believe no more than they ſee, is at preſent not much; however formerly ſolemnly, and for ſingular purpoſes, placed in and very near ſeveral of the chief Bowels: yet the ſumultous æſtuation which this Hypotheſis continually placeth nature under, like the meeting of two contrary ſeas, may be ſufficient, in reaſon alſo, to decry and diſcard it: for which reaſon alſo lateral homogeneal inoſculations of bloud-veins with bloud-veins, as they are frequently found, ſo they may as eaſily in reaſon be admitted. But the inoſculations in the *Q*ueſtion being final; I wave theſe that are lateral. And final inoſculations are on all hands, agreed to be in a manner impoſſible to be ſhewn; becauſe, when granted to be, yet there where they are, the veſſels are ſo minute, and ſlender, that it neither can be eaſy for the quickeſt ſight, nor the moſt cautious

hand

hand, either, without hurting them, to come at the veffels, where they are inofculated, or to diftinguifh the veffels when heterogeneal at their inofculation, or to difcern a final inofculation, whether of veffels of the fame, or of different kinds. So that this whole Queftion about final inofculations being to receive its decifion by the judgment of reafon; fuch final inofculations as are homogeneal, as of Arteries with Arteries, and Bloud-veins with Bloud-veins of the fame Trunk, fuch as in this Point were underftood and intended by the Antients, are no way capable of defence, allowance, or juftification at the Barr of reafon: for it experimentally, and to the eye, appears, that if any fuch inofculations fhould be fuppofed, the fame Humours at the fame time, from the fame place, fhould move two contrary ways, to wit, both to, and from the heart, and in veffels rifing from, or gathered at after into, the fame trunk: in which account, the lung vein, the hollow vein, and the port vein, though all of them bloud-veins, yet becaufe of the diverfity of their Trunks, are to be efteemed as heterogeneal. And finall homogeneal inofculations generally, and therefore alfo between the Mammaryes and Epigaftricks, being thus everted; the old doctrine oncerning the ground of the fympathy between the Breafts and womb cannot be retained.

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tained. And if any think by a short alteration
of, and putting heterogeneal inosculations
for, the rejected homogeneal, the ground of
this sympathy will remain firm and clear as of
old ; to wit by making humours to be trans-
ferred from the womb by the Epigastrick
arteries and mammary veins to the Breasts,
and by the mammary arteries, and Epigastrick
veins from the breasts to the womb ; I suppose,
that though such inosculations be not as the
other, impossible, though undemonstrable to
sense, yet upon mature considerations, these
will not be judged sufficient to found this sym-
pathy upon. If we lay the matter seriously in
the ballance, even such heterogeneal and
final inosculations do rather hinder the trans-
ferring of humours from the one part to the
other : to be sure, look how much falls, or is
any ways drawn into their capacities and ca-
vities, cannot by them be so transferred, but
is otherways disposed of : for the mammarie
veins carry not to the breasts, but to the sub-
clavians, and so to the heart, and the mam-
mary arteries carry not to the breasts, but to
the Muscles and parts on the forebelly. In like
manner the womb will appear equally uncon-
cerned in those veins and arteries : for the Epiga-
strick veins carry not bloud unto the womb but
unto their collateral Iliacks, and so to the
heart : and the Epigastrick arteries carry not
<div align="right">bloud</div>

bloud from the Iliacks to the womb, but to
the Muscles lying on the fore part of the belly.
And of themselves, neither mammary bloud-
veffels reach the womb, nor either of the Epiga-
ftricks the breafts: fo as , notwithftanding
any thing in thefe veffels, we yet feem to be
in the dark as to the grounds of this confeffed
fympathy. Not to add, that in many Animals
fome , and, in fome all , the breafts are quite
out of the way of the Mammaryes, on the fame
fide of the Epigaftricks with the womb , and,
on that fide, as farr removed from the capillar
terminations of the Epigaftricks and mamma-
ries, as the womb it felf. And profeffedly
I underftand not , upon what neceffity the vef-
fels founding , or contributing to the fym-
pathy of thefe two parts, muft needs pafs di-
rectly between the one part and the other; and
may not as well here, as in the fympathy of
other parts be admitted fufficient for this pur-
pofe, by their common concern and relation
to fome fpecial branch, or general trunk,
And veffells thus related, touching and termi-
nating in thefe two parts, are not hard to be
found in feveral kinds; as arteries and finews
to convey like matter to both thefe parts: veins
and finews to reduce what is improper, fuper-
fluous, ufelefs, or redundant in either, or meet
and apt to be transferred from the one to the
other: in which both cannot but be affifted by
the

the agreement of their similar attractions, and
that conformity of substance, which is either
constant, or at some special times between
some, or all of each of their parts : which as
it may be a ground of the maturation & exalting
of these parts together, so thereby the Bodies,
pores, and passages, of both, are alike open,
for the reception of like humours : and the
same conform substance may be the cause why
those vital emanations & irradiations which
the generative parts receive from the parts
which are principal ; when, I say, those vi-
tal emanations come to be remitted and reflc-
&ted back again from the generative parts to
all the rest of the body ; then, above all others,
the breasts gather into themselves powerfully
those forces of nature, which are so reflected
back again from the generative parts univer-
sally to the parts of the whole. And such a
course, as I have hinted, is both plain by
view, undeniably certain, and familiar in the
usual method of nature ; the breasts, above
other parts, being exonerated by the evacua-
tions of the womb and the evacuations of the
womb, more especially then of other parts,
being diverted by the excretions of the breasts:
so that these being the common and trodden
paths of nature, there cannot but remain an
open way also for the tranflation of other mat-
ters. And though by the Mammaryes and Epi-
gaftricks

gaftricks, whether inofculated, or not inofcu-
lated, I have above fhewn, that fuch con-
veyances cannot be made; yet nothing hin-
ders, nay 't is very apparent, that fuch tranf-
lations may be aptly enongh made, nay no
doubt are made, by the thoracick and prepa-
ring veffels; to wit, by the Arteries and fome
finews directly, as bringing humours directly
unto the parts, and by other finews and veins
more indirectly, as they are apt to carry, or
not carry away what has by other veffels been
transferred from one of thefe parts unto the
other. I had difcourfed thefe things fhorter,
if I had not been concerned to quit my felf of
rafhnefs, with which you had attainted me for
feeming herein to have made a defection from
the Antients. I fhould afsoon defend the
Moons Eclipfses to be a celeftiall fainting,
and planetary ficknefs, and to be cured by
Cymbals, Drums, Cornets and Trumpets;
becaufe the Antients thought fo. Yet for the
fame truths fake I account my felf bound to
juftify the old Aftronomers in the caufes by
them affigned of the feveral fhapes of the
Moon, as depending upon its mutuatious light,
the proportion in which it is received, and the
pofitions from which it offers its felf to our
view; all concerning the Earth in the fhapes
of its lucid parts, fave in its Eclipfses fometimes
when it is at the full, being the product merely
of

of ignorance and want of obfervation. I am
not of opinion with many, that there is but
only one fountain of light, which is the fun;
to which by imaginary refractions, and re-
flections, and ftrange labyrinthæan windings
they would referre all other lights; fending
the moft glorious of the celeftial Bodies a
borrowing to this fountain; while evidently
in thefe inferiour Regions, in the mineral,
vegetable and animal Kingdoms, &c. not to
fpeak of the cæleftiall flames, feveral fpecial
fountains of light have been, and are frequent-
ly obferved, though neither fo full, nor dura-
ble, and perpetual as thofe above. However
we cannot but confeffe the Sun to be the Foun-
tain of the principal Lunar light; as another
more faint light difcernable in all parts of its
body at whatfoever age, fo it be out of the
Combuftion of the Beams of the Sun, which
lye betwixt us and the Sun, or the Beams in
vicinity to them, cannot be denyed to be innate
unto it. And the roundnefs of the Moons
and Suns bodies, being evident to the Eye in
every pofiture and revolution, and apparent
in Solar and Lunar Eclipfes, befides feveral
other demonftrations of their rotundity; it
follows from hence by neceffary confequence
that the Moons body cannot be guilded all
over, at the fame time, with fuch luminous
beams as it borrows from the Sun: feeing in
any

any two fphæres ; in whatfoever pofition , the
one without the other , there is ftill a certain
part of the furface of the one , unto which with-
out penetrating its body from no point of the
other can be drawn a ftraight line , as is eafy to
demonftrate. And the magnitude of the bodies
of thefe two Stars , being confufedly unæqual
the one unto the other , and the Suns the lar-
ger ; it follows in like manner by the fame
neceffity , that though all the Moon cannot be
at once enlightened by the borrow'd Solar light,
yet at all times , more than half of the Moons
fphæral furface , is fo enlightened ; except in
thofe cafes when the interpofition of the earth ,
or other opake bodies , obftructs the folar ra-
diations from falling upon it. For if a lofser ,
and a greater fphære be put and adapted into
an Ifofcelary Cone , whofe Axe is perpendicu-
lar to the circular Bafe at its Center , the Lines
in which the furface of the Cone toucheth the
furfaces of the fphæres , fhall be two Circles
lefs than the refpective greateft Circles , each
of its own Sphærs as is not hard to demonftrate.
And it is as apparent , that the Iforrhopi-
cal center of that part of the Moons furface ,
which is fo folarly enlightened , is ftill the
point directly obverted to the Sun ; thorow
which point a ftraight line drawn between the
centers of thefe two Stars pafseth. So as the
enlightened part of the Moon being ftill to-
wards

wards the Sun, no wonder, if in their con-
junction the Moon cannot be seen at all; as
chiefly becaufe of its combuftion, fo alfo be-
caufe the enlightened fide of it, is turned away
from us towards the Sun : that though, by
defcending into deep pits, Stars above the
Horizon may be feen at Noon day, yet the
Moon in this cafe cannot by fuch helps be in
like manner feen, its dark fide being to-
wards us, and the moft that our Eye can at
any time at one view fee of the Moons fur-
face, being ever lefs then the half of it,
as is demonftrable. Upon the fame grounds,
at the Oppofition, i. e. full Moon, the
enlightened part of the Moon as it is to-
wards the Sun, fo it is towards us alfo, from
whence follows, that all, that we can then
fee of the Moon, is enlightened, and that
much more of it is enlightened, than it can
be poffible for us to fee. In the Quadratures,
whether of increafe, or decreafe, the ftate and
pofiture of the enlightened, and unenlighten-
ed parts of the Moon, in refpect of the Sun
and us, offer themfelves in a lefs entire, divi-
ded, middle manner : to wit, the parts of the
Moons furface obverted unto us are but, as it
were, half obverted to the Sun, and the parts of
the Moons furface obverted to the Sun are but,
as it were, half obverted to us : fo as what is
obverted to us in the Quadratures is, as it
<div align="right">were</div>

were, half enlightened, and half unenlightned and at other not Cardinal times, the nearer that the Moon is to the full, so much the greater part of the Moons surface obverted to us, is by the Sun enlightened: and the nearer the Moon is to its Change, whether past, or to come, so much the lesser part doth the Sun enlighten of the Moons surface obverted to us. And that these things are really so, may be from hence confirmed: at whatsoever Age of the Moons crescency, or decrescency, if you observe its luminous parts seen by us, however horned, or bunched, the Sun is still on the luminous side of the Moon. Now the cause, why the Moons luminous parts appear plain and flat, rather than bearing out, like a Sphæral surface, or a portion of a Sphæral surface, is because the inequality of the visual beams, between the Eye and the several luminous points, is not so proportionate as to be discernable by our ordinary sight: for without such inequality, to be discerned in the visual beams, neither sense, nor reason can conceive true Idea's of a spheral surface. And upon these Hypotheses and principles all phænomena's concerning the Moons shapes, as to magnitude, proportion, continuance, situation, alteration, alternation, being perpetually and accurately solved; the Novelty which

would

would dethrone this doctrine so apparent, so rational and unconvincible of the Antients, as it is unworthy the acceptance of the confiderate and judicious, so it cannot but be unmeet for them to impofe and obtrude upon others; to omit the groffe inconfiftency involved in it, which I fhall in my next at large unfold, if thefe things fall fhort of giving Satisfaction.

The

The third Anſwer.

That there are freſh waterſprings at the bottoms of the moſt
antient ſeas. Whether Mercury *by frequent tranſhaping*
it ſelf, and often reduction looſe not ſomwhat of its
powers and virtues ? That the Doctrine and being of
the four Elements, as unmixed bodies by their mixture
making up all other bodies, is not unqueſtionable.
In the four Elements may be allowed to be the general
and moſt common lodges of the Worlds firſt Elements ;
but themſelves at the moſt can but be allowed to be only
ſecondary Elements. How nature may ſo ballance the
firſt or ſecond Elements by ſome ſpecial Symbolical pro-
perties among them, as to elude all the endeavours
which art can poſſibly make for the bringing of the firſt
Elements to view and light. That the number of the
four Elements and of their properties or qualities ſingly
or in Conjugations aſcribed to them ſeemeth inſufficient
to veſt them in the right of firſt Elements ; upon the
knowledge of whoſe natures, all Phyſical phænomena
ſhould be capable of explication. That the antients ſeem
to have allowed a greater number of firſt Elements. A
conjecture what the antients might rationally at firſt
deſign at their firſt introducing the Doctrine of the four
Elements.

SIR.

THat freſh water ſprings lye at the bottom
of the ſeas, both frequent, and with
aſhing iſſues ; is to me upon ſeveral obſerva-
ons, not undiligently made, as abſolutely

E   cer-

certain, as to you it feems impoffible : and this
I judge not only in the feas, which have made
inchroachments by inundation upon the antient
bounders of lands, which before were plen-
teoufly up and down watered with fprings;
but as well, there being the like reafon, in
thofe feas, which are able to plead the higheft
and moft antient præfcription, and cannot be
any other wayes chargeable, then in their
dayly fluxes and refluxes, with the leaft new
invading of the earths bofome, and of thofe
wells of coldboiling-natural nectar, with
which it is ufually there ftored. And why
fhould natures opening a vein of frefhwater
into the fea, feem fuch a fea monfter when at
land we ordinarily meet with divers fprings, of
different virtues and originals, meeting at after
together in the fame channel ; from their con
courfe and mixture conceiving fecret virtues
manifeft alterations, and fpecial properties
as ftrange to the illiterate and unexperienced
and generally wondered at by the moft, as th
boiling heat appearing prefently in the fud
denly mixt Oyles of Tartar and Vitriol, thoug
cold, when poured together. I fhall not urge
that fome, would have the feas, proportio
nately to their depth, frefh at the bottom.
only move, if this be not more eafy, to be af
fented to, then what is dayly feen, and there
fore not to be queftioned in point of truth

tha

that the sea fishes &c. Though continually living in that briny pickle remain still however fresh: many times the sea-fowle, that mostwhat fly but about it, sometimes swim in the surface of it, senting and tasting much stronger of the sea, than the fish that live deep and constant in it. But Experience being the grand Umpiress in the Question, in assurance of its convictiveness, I forbear at present. For your reducing of *Mercury* after all operations of fire, whether actual, or potential upon it, I have no reason to cherish suspitions of the truth of such performances: but that reduction shall be so perfect, as to give back the *Mercury* as absolute in all virtues, as it was, or could at first be delivered, will not be easily consented to, by those that know that fire burns the chief wing upon which in Amalgames *Mercury* carryes along with it Gold: and the force of fire upon it, is in the nature of a rape robbing it of that virgin treasure, to which the noble mettal is so sequacious; which once lost, can never be restored again to the defloured *Mercury*: as in all volatiles that which is of nimblest wing flyes first, and the highest spirit first: and when an impregnation is to be made by fire the work of Philosophy is judged chiefly to lye in the governing, of the firing, that, like the Sun, it may give enlivening heat, and not become, as a destroying Element. The reason, why so

E 2   few,

few are acquainted with the excellency and prærogative of this pure *Mercury*, is becaufe generally it is a fire-burnt *Mercury*, which is at firſt delivered to us. I omit that in a thouſand inſtances, after Art has ſeparated the natural union of parts, though it may again unite them, yet that re-union will in many points fall ſhort of the firſt natural union. But why does it ſeem ſo monſtrous to you, to call in Queſtion the exiſtence of the four Elements, it being a doctrine, which in all Ages has been attended with doubts? if you think you can ſhew as *A*ir, and Earth and water, yet what ſhew can you make of Elemental fire? And ſince the diſſolution of the Orbes, there being no Concave of the Moon, what region is deſigned and intended for it? I preſume both Hearths, and Altars, are too low, to be the proper Sphære of this high Element. And upon the whole, as little can be ſaid for any of the other three as for this: the State of the *Q*ueſtion and controverſie being rightly underſtood. For it is not fire that is denied, or Earth, or Ayr, or water, but all as Elemental. How great a ſhare and proportion theſe have in making up this part of the worlds fyſteme, is too evident to ſenſe to be called in Queſtion: many leagues of Earth, or ſea, anſwering to each ſingle degree: but that theſe four are the firſt uncompounded compounding

bodies

bodies, of which all others by their mixtion confift, and are made; feems, from hence, to follow by a very fickly confequence. Not without folid confideration, and fufficient caufe, did the Antients honour the Earth with the ftyle of the All-feeding-Earth : what innumerable vegetables and Animals fpring, live and grow in its bofome, not to touch at the ineftimable treafures lodged in its bowels? Others, and of the worlds fages, with honours not unequal, affign unto the water not only Beauty's birth, but the Rife and original of all things: and every moment convinceth the ufe and neceffity of Air &c. Thefe are Arguments of their univerfal ufe, concern and excellency, but do not prove their Elementalnefs, no more then that one Tree is the Element of another, becaufe the one is engrafted into the other, or that the mother is the Element of the child, while it lives annexed unto her in the womb. That in thefe are the general and moft common Lodges of the worlds Elements, from which each nature may furnifh it felf with what is convenient for its being, nutrition and growth, may eafily be confented unto : but that thefe are the very Elements unmixed and uncompounded, will be an Herculean task to make forth by any Argument, or experience, fo long as each in its region appears replenifhed with all variety of Beings. What a nume-

rous

rous diverfity of Earths are to be found in the
Earth, of diverfe colours, weights, virtues ;
fome healing, fome fcouring, fome binding,
fome Alexipharmical, befides chalkes, and
Marles, and feveral clayes, fands and gra-
vels, noble quarries, rich mines, coals,
bitumes, marcafites, falts, minerals and me-
tals ? fo in water, all coagulable vapours and
exhalations meet condenfed with the alluvion
and diffolution of various falts and other mi-
nerals, together with manifold fubterraneous
Oyles and fpirits ; whence the wonderful dif-
ference of their weights, fents, colours, tafts,
confiftence and operations. In like manner the
Air, what is it but an uncertain, unconftant,
randome compofition of all forts of fumes and
vapours, according to the nature and pofition
of the Atomes difperfed in it fometimes clear,
fometimes cloudy, fometimes healthful, fome-
times peftilent, fometimes delighting, at other
times offending the fenfes ? And as difficult
will it be to find and fhew any uncompounded
Fire. But you will fay, that though thefe be
not Elemental Earth, Fire, Aire and Wa-
ter, it hinders not but there may be fuch Beings
and bodies ; I anfwer, I have no abhorrency
againft the opinion, if fo by any practice, or
in any diffolution thefe Elements could but be
fhewn, and their fufficiency to explain the phæ-
nomena of nature : but thofe parts, which, ge-
nerally

nerally in diſſolutions: are offered to be accepted for theſe Elements, appear nothing leſſe, and, beyond all diſpute, very diſtant from uncompounded natures; and the analogous parts in ſeveral diſſolutions, as different one from another, as one of the Elements can be from another Element; which is repugnant to the nature of firſt Elements; as is uſually and truly urged againſt ſalts, ſulphures and *Mercuries* to put by their claim in firſt Elementarineſſe, in the mixtion of bodies: *Viz.* that in ſeveral diſſolutions, the analogous parts anſwer not in the leaſt one to another, except it be in ſome very general, and external conformity; but one is ſowr, another ſweet, another ſharp, another fælid, another fragrant, thus expanding into all manner of variety, wherein ſhould be nothing but the pure ſimplicity of a firſt Element. In a good ſenſe, and with fair Explications, I have been ever ready to acknowledge their intereſt as ſecondary Elements, as a doctrine conſentaneons to the Sacred Traditions of Theology; or as before, as the principal Lodges of the firſt Elements: but neither that which fils the regions, nor that which remains after their diſolution of mixed bodies, could ever yet to me ſeem capable of the denomination of firſt Elements. And poſſibly it may be one of natures myſteries never to per-

mit

mit us to fee the firft Elements naked fo cu-
rioufly ballancing them in their connexions,
that all attempts of Art, for their difcovery,
fhall be eluded : fo any Artifice to refolve and
anatomize a natural body into its firft princi-
ples, grounded upon the fixednefs and vola-
tility of the parts, is eluded, if firft, or fecond
Elements of the fame fixednes, or volatility
be connexed : fo if grounded upon difsoluble-
nefs in a certain Menftrue, it is eluded by con-
nexing feveral firft, or fecond Elements æqual-
ly difsoluble in that, or the like Menftrue : fo
corrofious by feparatories, fumigating, anoin-
ting, cementing are all eluded as to the bol-
ting forth of the firft Elements, if they meet
with feveral firft, or other Elements æqually
paffible in thofe operations. And the very
number of the Elements has not feldom ren-
dered them unto me very fufpitious, that they
could not be firft Elements. For by what fair
Argument fhal the Quaternion of them be de-
monftrated? if, as is ufual, we argue from
qualities, whether fingular, or in fymbolifmes,
I fee not how this number can be maintained:
more qualities, and more conjugations of
them, ought by the fame reafon to prove a
far greater number of Elements. And alas,
how vainly light do they render themfelves,
that, by four conjugations of difputable
qualities, feek to folve all the phænomena
which

which are in the world ; of the Loadstones
pointing northward , its drawing untoucht
steel , the Load-touched steels drawing of
other steel , the bleeding of the slain at the
presence of the Murtherer , the moving of an
untouched Lute-string being Unison , or in a
strong consonancy string that is moved , the
sinking of persons drowned the first dayes ,
and then floating to the top at after , they
have layd soaking so long in the water , the
Cramp fishes astonishing the Fishers hand at
the long distance of line and Angle , the
strange Ebbings and flowings of particular seas
and springs , and a thousand other ? Upon the
old principles , how lame is the doctrine of so-
lidness and fluidness , of opacity and transpa-
rency., in bodies sometimes of the same so-
lidnes , sometimes of the same fluidnes , be-
sides a million of other instances , whose reso-
lution mateth the doctrine of the four Ele-
ments ? However mis-understand me not , as
if I denied Elements , or first Elements : these
must either be allowed , or no mixtion ; of
which we have lucide examples in every cor-
ner of the world : for it was a Golden-rule in
the school , which now I am opposing, that
the same, or like, as so, working upon the
same , or like, as so, cannot make any alte-
ration : that I suppose I have good reason not
to be forward to embrace the new received
<div align="right">opi-</div>

opinion , fo much cherifhed by fome perfons of Eminency and parts , that there is only one Element of all things licked into feveral external fhapes and forms. But this controverfy not concerning your judgment , it were trouble now to purfue an impertinent. The reft I fuppofe , any where defenfible , fave at Stagira ; where Fire muft burn , Air poyfon , Earth bury , and water drown , whatfoever fhall be fufpected in , or alleadged againft the Traditions of the imperial Philofopher. However my thoughts are ftill comported to yield up themfelves to that doctrine upon the firft clear and experimental demonftration of it ; if any fuch might be hoped. In the interim I want not fair footfteppings in the Antients to induce me to think , that when they make Bodies to be compofed of that which is hot , and that which is cold , of that which is dry and of that which is moyft ; they mean nothing lefs then thefe four bodies, or rather vaft Amaffements , vulgarly now known by the name of the four Elements : but they rather intend , the true Elements of nature , it may be feldom , or never feen alone ; in which , fuch qualities , or properties are fpecially eminent : and they are fo far from reftraining natural compofitions to the at prefent received Quaternary of Elements , that conftantly at the fame breath they hint many other :

ad-

adjoyning in their recitals of natural mixtions, temperaments, and compositions, to that which it hot, cold, moyst, dry, Elements endowed with several other properties, as that which is salt, that which is sowr, that which is bitter, that which is sharp, that which is sweet &c. and the extremities and distempers intolerable unto nature, whether in general the Animal nature, or more especially the Humane nature, are made to be, when such Elements sever from their mixture, and go apart by themselves to violating, and breaking up that sweet proportionate composition and harmony, which is so friendly and grateful unto nature. As the Pyrotechnists analyse bodies, upon the consideration of several differences and contrarieties which are in their compositive parts, as that some will abide the fire, others not; some be wrought upon by such dissolvents, or corroders, others not: so the Antients to reduce all inanimate things to four heads, may have insisted on several contrarieties, paving the way to such a doctrine: *Viz.* that such bodies are either firmly solid, or else fluid: and those that are fluid, are such, either in a grosse corpulent matter, or in fine exhalations: and those exhalations, either apt to conceive a flame, and burn, or else of a nature, thereunto indisposed. And according to such a distribution, many various bodies
fall

fall aptly enough under the fame head of Amafs-ment: and all inanimals in the world may be eafily reducible to fome one of thefe four heads. But then thefe four heads are not parti-cular firft Elements, uncompounded, and fo conftitutive of things in mixtion ; but general notions, unto which the inanimals of this lower part of the univerfe, are aptly reducible. Favour thefe Conjectures, which obtrude no-thing, only labour what they can to juftify the methods of the Antients. Send back my in-ftruments and the laft tranfmitted Problemes with your firft convenience.

The

The Fourth Anſwer.

Concerning the ſeams of the Skull, and the number of the ſeams by which the Skullbones are joined one unto another, over and beſides the three principal proper and true ſeams. That without contradiction, variety may be admitted both in the number and order of ſuch ſeams. That neither abſtruct nor concrete Mathematicks are wholly exempt from a liablenes to ſuch varieties. That the pre-eminence of right-ſide parts is no way natural in ſuch as twin but only at pleaſure.

S I R.

THe learned men were riper for controverſy, than became the gravity of their profeſſion, or elſe upon the Point, which you have tranſmitted to me, held by each with ſo little real, though a manifeſt nominal difference, it had not been eaſy to have occaſioned the hot language contained in your report. Though the one make but five pairs of ſeams, by which the Temple bones are joyned to the wedge bone, and alſo to the two Crown bones, and thoſe (beſides the three principal ſeams) to the forehead bone, or bones, and to the Nowl bone ; and the other, contrary to the general doctrine of the Anatomick School, aſſerts that there are fix ; and both make their appeal to Autopſy and the Skeleton ;

I ſee

I see not wherein either is difabled to maintain his Affertion, as both agreable to truth and juftifiable upon the view: only not without fome rational ground for the difference, *Viz.* either hath, as I fuppofe, his different method, upon which he infifts in making up the account of the feams. For the number will differ according to the ground by which you will diftinguifh and make out the unity of each feam: as alfo according as the feams are accounted fingly, or by pairs; from which laft manner of accounting may arife alfo a third different number of them, though ftill without any change in the thing and object. Not being privy to the terms upon which this controverfy was between them managed; this Reconciliation, though unqueftionably true, and pertinent to the Queftion, as by you in general propofed, may however poffibly be inpertinent to it, as by them ftated and defended: though I underftand not upon what other terms the Affertions of both can be capable of juftification: and I am not willing to think otherwife, between perfons, fo eminently learned. It cannot be denied but in thefe nicer things obferved in the head, and diftinguifhed ufually by numbers, as the feams, and holes of the fcull, there has not wanted fome feeming diverfity and obfcurity: while it has been thought, that merely at the pleafure of Authors

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thors and of thofe that fit in the Anatomical
chair, fometimes one thing, at other times
another, and not conftantly the fame thing,
is made to give the order, unity, integrity and
diftinction of feams. So the Nowlebones con-
nexion with the Crown bone, and Temple
bone of the fame fide, is generally conftrued
to be two feams, and of two diverfe kinds;
the one true, the other meridous, or defective:
though fo fair be the continuance of the one
feam unto the other, that the one is ufually
known by the name of an Additament unto
the other: and yet in other feams diverfity of
bones doth not, with all, make multiplicity
of feams; as in the Lepidoidal, or fcalefeam
where the Templebone is joyned to the Crown-
bone and to the wedge-bone by a future, gene-
rally accounted but for one: fo the connexion
of the inner procefs of the forehead bone, to
four feveral bones of the upper chapp, is by
moft accounted but for one common feam.
The like, or worfe perplexities occurre fome-
times in the account of the holes in the head;
while fome of the holes, which are common
to feveral bones are accounted in each bone:
others, in like manner common only in one
of the bones, whence many times the number
of the holes pertinent to each bone is left con-
fufed, and imperfect. For the reverence I bear
unto the great Artifts, in whom thefe things
may

may be ordinarily obferved; I am not willing
to impute them any otherwife then to their
great intentneffe upon matter and things; by
which they feem overborn into a certain neg-
ligence of names and appellations. And thofe
whofe ftudy is rather to be Anatomifts, then
fo efteemed, and draw their Inftitutes and
obfervations from the Body, more then out of
Books, will eafily confent with me that in
this whole Doctrine, of the feams, and holes
of the head, names and numbers are of lefs
moment, fo there be a clear underftanding
of the things themfelves. So none, who has
truly made ufe of his own Eyes can deny
fome parts, or fides of the true feams of the
skull to be mendous, and fome parts, or fides
of fome of the mendous to be in a fort true : that
in feveral of them the diftinction of true from
mendous, and futures from harmonyes, feems
to be oft more matter of name impofed at plea-
fure, without clear and fufficient ground for
real and peremptory diftinction. Things thus
ftanding, in the conteft you mention ; how
eafy is it to make, on the fame fide, the feams,
by which the Temple bone and wedge bone
are joyned one to another, and (as before) to
the Nowl bone, Crown bone and Forehead
bone, to be at pleafure, either five in number,
or if you pleafe fix ; For the one, or the other
number follows, according as you fhall take
the

the connexion of the Temple-bone with a
Crown-bone and the wedge-bone , to be one
feams as it is moft ufually accounted ; or·elfe
to be two ; for which , being the connexions
of two and two feveral bones , there wants not,
as appears , fome fair and probable ground.
And thus though I would not charge with
falfity , that whofe truth is from real grounds
fo plainly defenfible , howbeit fomewhat
varying from the ordinary methods , and fy-
ftems of Authors; yet I judge it not meet
we be too forward , by reafon of our more
clear infight into things , in altering , flight-
ing and rejecting the Terms , names , diftin·
ctions and methods , which have , time out of
mind , been in continual ufe with learned men.
For names , that were firft coyned for diftin-
ction and inftruction , if they fhould be dayly
and hourly changed , would be fure to intro-
duce confufion. And though in Names and
numbers the Antients have ufed fomewhat of
authority , they have notwithftanding , in
the fame fubject , fairly exercifed their reafon.
So , with a very little allowance , this whole
doctrine of the futures , has been caft by them
into a Mould methodical enough : the fum
whereof in few may be this. The futures of
the head either concern not, or elfe concern
the bones of the upper chap : thofe which con-
cern not the Bones of the upper chap , are

F   either

either largely toothed futures, *Viz.* the Crown
future, the Dart future, and the Lambda fu-
ture; which may also receive another ground
of diftinction from their concerning the two
Crown-bones connexion one with the other,
or with the Forehead bones and the Noul-bone:
or otherwife; other futures, not concerning
the bones of the upper chap, are not fo fairly
not deeply, fometimes ftarce at all, at leaft not
fo manifeftly dented, or not in the fame man-
ner; as the Lepidoidal connexting the Tem-
ple-bone to the Crown-bone and to the wedge-
bone; the connexion between the Crown-
bone and wedge-bone; the connexion between
the Noul-bone and Temple-bone, commonly
called the Additament of the Lambdoidal;
the connexion between the Nouls additament
and the wedge-bone; and the connexion
between the Fore-head bone and the wedge-
bone: which may alfo receive another ground
of diftinction from their concerning the con-
nexion of the Temple-bone aud wedge-bone
one with the other and with the Noul-bone,
Crown-bones, and Forehead-bone. So the
futures, which concern the bones of the upper
chap, either concern fome one, or more of
them in common with fome of the aforemen-
tioned bones; or elfe they concern, the bones
of the upper chap alone, as connexed mutually
amongft themfelves. The futures which con-
cern

cern some one, or more of the bones of the upper chap, in common with some of the aforementioned, make connexion only with some of these, *Viz.* the Temple-bone, wedge-bone, or Fore-head bone; for the Crown-bones and Noul-bone are no where connexed to the bones of the upper chap: And they are particularly these; *Viz.* the connexion of the outer processe of the Fore-head bone, with the inner processe of the first bone of the upper chap: the connexion of the first bone of the upper chap with the wedge bone: the connexion of the inner processe of the Forehead-bone with the fifth, fourth, second and third bones of the upper chap, being in the order recited, connexed from within outwards: the connexion of the outer, or yoke processe of the first bone of the upper chap, with the yoke processe of the Temple bone, in the middle of the yoke bone: which yoke bone is not a distinct and several bone, but made up of two processes of two several bones: and the last is the connexion between part of the upper, part of the difsepiment and the wedge-bone, Lastly the seams which concern the bones of the upper chap only among themselves are the connexion of the first and fourth bone, the connexion of the second bone with the third and fourth, the connexion of the third and fourth bones, the connexion of the fourth and fifth

F 2   bones

bones, the connexion between the two fifth
bones, the connexion of the fourth and sixth
bones, the connexion between the two sixth
bones, the connexion of the other part of the
upper part of the diffepiment with the fifth
bones, the connexion of the lower part, of
the diffepiment in the Plough-fhare bone with
the fixth, *i. e.* the palat bones and some fmall
part of the fourth bones. In all which, if
fomewhat at pleafure, they have ufed authority
in the numeral order of the bones, and in the
account and order of the feams, it is no more
than is freely allowed to each Artift in his
Art, fo long as the Authority they ufe, neither
bears, nor creats any repugnancy to the fubject
matter. And fo even in abftract Mathema-
ticks, fomethings are determined, not by ne-
ceffity, becaufe they can be no otherwife, but
by long ufage and authority, becaufe they may
conveniently be as is determined. So Arithme-
tical numerations, which are every where re-
ceived and taught in Decuple proportion;
with as much facility and Mathematical ac-
curatenefs might be inftituted in undecuple prc-
portion; and then the ufual Probes, fhould
not be by cafting away of nines but of tens;
or in fexage cuple proportion; and then the
Probes fhould be by cafting away of fifty nines:
or non cuple proportion, and then the Probes
fhould be by cafting away of eights. And the
like

like inftances might be given in Geometry; and in feveral, if not moft parts of Concrete Mathematicks. So in ordering the Account of the Brain ventricles, of the fides of the fame bones, and fometimes of the branchings of finews, veins, Arteries, other veffels, and the like; cuftome, and authority, commonly gives præcedency to the parts, cavities, or branchings, pofited on the right fide; though without offence unto Truth, all Accounts might as exquifitely be compleated in a quite contrary order. But though thefe things might be, and as well, as what is; yet what is, being as accurate, as what otherwife might be, it is fufficient to know that thefe things might be otherwife methodized, without renouncing the Antients method. I have, you fee, too high an Opinion of the perfons concerned, to imagine their difference could arife from, fome peculiarities in unufual skulls; in which it is not uncommon to find great difference in the numbers of bones, feams and holes, not only under diverfity of Age and fex, which, is fo common, it is not otherwife to be expected; but as well when no fuch caufes can be alleadged, for the diverfity: nay, many times in the main and principal, commonly called True feams we do not ever find a conftant regularity and uniformity. But of thofe things, which

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feldome are, we are not to maintain Controverfy, as if they were the way of nature. This, Sir, I have written not to Inform, but confirm your judgement; which I know fo well verfed in the Syntaxe of this our humane body, that it cannot diffent from what is.

The

The Fifth Answer.

That immense volatility may consist with immense ponderousness. That tincturs may be altered by maturation without any addition, Whether the appearing and motion of comets may be before their appearance predicted. That no such particular predictions can be made concerning the meteors which are in the Atmosphear of the earth, nor of the first appearances of such Comets as are supposed to have their Original from new amassements of Cometical matter in other Atmospheares; though after their first appearance, upon some observations accurately made, somewhat, though nothing so peremptorily as in other cases, may be predicted, relating to their future motion. That it is not impossible, but concerning Comets which are permanent bodies, and not new amassements, predictions may be made long before, of their future appearances and motion.

SIR.

I Take your satisfaction, upon my last proposed, clear, and doubtless Experiments, now since by you proved and approved, as a fair acknowledgment of that Truth, which however to me upon its former evidences needed no farther confirmation : *Viz.* that so different are the genuine notions and qualities of fixednes and gravity, that immense volatility may and is ordinarily consistent with an

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immenfe fpecifick ponderoufnefs; arifing not
from the moles and quantity of the bodies un-
der confideration, but rather from their na-
tures and kinds. That what in your firft veli-
tation you affumed as abfurdly groffe, incon-
fiftent, and impoffible ; is now, upon your
own acknowledgments, moft eafily and ob-
vioufly demonftrable by experience. No lefs
certainly is to be acknowledged, in what you
propofe for the tingeing of metalline bodies,
only by ordering and attending them in the
management of their maturations, without the
addition either of body, or fpirit, as being
all the time of this operation under the fure
feal. Could I perfwade my felf it fhould not
be refented as a grand unkindnes, to be fi-
lent in what you call for my thoughts in, in
the clofe of yours ; *Viz*. Whether prognofticks
at certainty, fuch as are of Eclipfes, of Coi-
tions, oppofitions, motions of other Starrs,
whether, I fay, fuch prognofticks and of like
certainty, may not alfo be made of Comets,
of their Appearances, common impediments
removed, and of their motions; I fhould, if
it might be, herein, lay *Harpocrates* finger
upon my lip, and feal them up into a perti-
nacions filence. Not but I am defirons to know,
andwilling to fearch after truth : only I fear
me, thefe are fecrets of nature, by their pecu-
liar myfterioufnes, finking themfelves fo low
into

into the pit of obfcurity , that the ftock of obfervations and difquifitions about them , which the world as yet hath , is not able to raife them fo high , and place them fo near day , as to be within humane reach and difcovery. What I now offer is a Cæfarean birth of the mind , not brought forth by me , but cut out of the womb by your importunity : if it want fhape , licking and lineaments , accept it as an unripe Abort , and either hatch it to perfection in your Thigh , or give it a little duft to cover it. I take by way of præfumption , that in this Quæry and Queftion you mean not by Comets , any of thofe more ufual , lefs permanent is regularly moved Meteors , bred , gathered , fired , and burning in the Atmofphære of our Earth ; after their Appearance , there cannot be much certainty of their Motion , though fometimes there may be conjectures probable enough , and in the Event by obfervation juftifyed , when the fuel , or fovent matter of fuch Meteors , is manifeftly , upon what account foever , known to be only , or moft copioufly fituate and difpofed fome one particular way : but of the generation and firft appearance of fuch Meteors , particularly the moment when , and the point , or exact place where , they fhall appear , much lefs of certainty , in fuch pronofticks is to be expected ; there being fo much variety , contingency , and

uncer-

uncertainty, in the caufes and meeting toge-
ther of thofe caufes, which contribute to their
production. And though there are, and have
been many prædictions in general, and Rules
of prædicting Aftrologically, or Phyfiologi-
cally, from the feafons of the year, the
temperature, and diftemper of feafons &c.
Concerning fuch meteors in general; yet in a
matter fo unallyed unto certainty, replenifhed
with all manner of cafualties, to promote, or
retard fuch productions, I have not known
any offer peremptory Pronofticks of the kind
of the meteor, its fhape, magnitude, dura-
tion, motion, with abfolute determinations of
its time and places, at any time beforehand.
The quality and condition of the fubject mat-
ter, making it as impoffible to bolt out fcien-
tifical and oracularly certain predictions of
fuch meteors, as it is, a year before, to prove,
or fhew that in fuch an hour, and in fuch a
quarter, fhall be a Rainbow, fo colour'd, fo
continued, or difcontinued and of fuch limited
dimenfions: or that, fuch a moment, from
fuch a point of fuch an Azymuth, fhall a de-
volant Star fpring forth, which, in its fall,
fhall run obliquely thorow fuch and fuch Azy-
muths, and expire at fuch an height above the
Horifon; or that fuch an hour, in fuch a
Longitude and Latitude, fhall in fuch Altitudet
and pofitions, and of fuch dimenfions, be
                                              feen

seen by day four Suns , or by night three
Moons. To lay aside therefore the considera-
tion of these , as supposed impertinent to our
present purpose ; what may judiciously be con-
cluded upon your *Questions* concerning those
other Comets , which lye without the compasse
of this Earths Atmosphære ? If there be not two
sorts of such celestial Comets ; there are at least
two several and very different Hypotheses and
notions under which they are considered by Ar-
tists : and Artists of equal worth and fame, order
their reasonings , some upon the one supposi-
tion , some upon the other : *Viz.* Some , as if
Comets were new made bodies , amassed , and
gathered in some of the superior Atmosphæres ;
many of which are , not without fair reason,
supposed to be in those vastly remote æthereal
regions : others, as if Comets were coæval to, and
neither less permanent , nor more new, then the
rest of the Stars ; only seldome seen, and when
seen, soon passing again out of sight , by reason
of the Line, upon which their Center is moved :
and nothing as yet appears hindering the truth
of the possibility and consistency of both these
Opinions ; leaving it especially indifferent in
the later Hypothesis , to call such stars at plea-
sure by the name , either of Comets , or New-
stars , or rather seldome appearing Stars.
Comets upon the first supposition , seem not to
want some affinity with several , especially of
<div align="right">the</div>

the more eminent, meteors of our Atmoſphære:
yet allowing a vaſt difference between them
in place, proportion, duration, motion, and
the like circumſtantials. And as our Globe,
though in its ſelf large, is but a minute thing,
compared with many of the cæleſtial Bodies;
that the vaſt error of the Earths ſemidiameter,
by taking the Earth for a ſmall point, many
times in Aſtronomical, Gnomonical, and
other operations, creates not by conſequence
any perceiveable Errour: ſo it is not unlike,
but the Atmoſphære of our Globe is vaſtly diſ-
proportionable to the Atmoſphæres of many
of thoſe heavenly lights: which may in a ſort
be gathered alſo, from the ſmall power, which
our Atmoſphære has in the preſſures which it
makes upon moiſt and liquid bodies: as alſo
from the common vaſtnes of cometical bodies,
frequently accepted as much larger then all
both our globe and its Atmoſphære, and there-
fore requiring a proportionately immenſe
place both for their being and motions. And
that ſuch huge dimenſions are not groundleſly
attributed to the cometical bodies, is rendered
undoubted from the glorious and glaring light,
with which eminently above moſt of other
Stars, they dazle, to admiration, the minds and
Eyes of mortals at ſo great diſtance, that ſome
times little, or no Parallax can be obſerved in
them. As to theſe therefore to ſolve the Que-
ſtions,

ftions, unto which you have tasked me ; I
cannot but judge it improbable , with any af-
furance to make prædictions before hand , that
in fuch limited moments of futurity, fhall appear
fuch Comets , as are to be at after amaffed ,
gathered , formed , and embodied in the At-
mofphæres , which are above : if feeming no
more wilde to give heed and credit to the fu-
perftitious Auguries and foothfayings of the
Antients , then the moft fubtle reafonings upon
which fuch predictions can be made , in a fub-
ject of this nature. Yet though their firft emer-
ging and appearance cannot upon any calcula-
tion be foretold ; however at after they have a
while appeared , and fome , though not very
many , curious obfervations have been made
of the changes of place , and other circumftan-
ces , which they make in their feveral appear-
ances , at fuch and fuch exalt diftances of time ;
it is fairly enough probable , that often , by
an ingenious prognoftick , the courfe of fuch
a Comet may be nearly traced , and fet down
before hand , as to way and fpeed , only with
this limita tion , if the fubftance of the Comet
do not firft vanifh , or diffolve , or by reafon of
its inftant diffolution divert from , haften , or
retard its courfe : upon which fcore of their
diffolublenefs , and therefore their ere long to
be expected diffolution , no prognofticks, upon
them and their courfe , can be fo abfolute and
per-

peremptory , as thofe which may be made con-
cerning fuch Comets , as are properly couched
under the fecond fuppofition. And the grounds
why moft what in thefe Comets of the firft con-
ftitution , or fuppofition , from paft obferva-
tions , a fair conjecture may be made of their
future courfe notwithftanding their new pro-
duction and amaffement in Atmofphæres , are
chiefly hereupon founded: that fuch cometi-
cal bodies , as they are demonftrably vaft , and
manifeftly , and by their nature diffoluble , fo
their difsolution is not obferved very momenta-
neous, but moft what more , leifurely by degrees
and in time; in fhort , they flame forth of a
fudden , but expire gradually ; fo as to be able,
for a while , to retain the impreffions of regu-
lar motion, whether communicated from fome-
what in the Center of the fame Atmofphære ,
or from the confluent efficacy of feveral
Agents all contributing conftantly , each ac-
cording to its nature , to the Comets alone ,
or its and its Atmofphæres motion , whether by
æquilibration , pulfion , traction , prefsure ,
manifeft , or fecret ; or elfe inwardly wrought
into the very fubftance and nature of the
Comet: efpecially the Comets place being in
thofe lofty æthereal regions , where are
perpetual circumgyrations and every motion
that is obfervable , the more accurately it is
obferved , it is found the more regular. And
if

if it be urged here-against that the motion of
Comets feldome, or never anfwers the motion
of any other Starre, but commonly has a crofs
trajecting motion, whereby it moves afloope,
athwart, tranfverfely, or obliquely,varying from
the ufual, formerly obferved motions. As this
cannot be denyed, fo it doth not at all infringe
the poffible regularity of fuch motions, which
are obferved to be moft what in the fame plain,
though oddly pofited: and that there are many
and, diverfe fuch undreamt of motions in the
Heavens, conftant and regular, of which fave
by fome fuch, or late difcoveries, no hint hath
as yet bin made forth unto us, may in as good
reafon be allowed, as that vaft number of Stars
in the skyes acknowledged, of which, with-
out the Telefcop, 'tis impoffible to have any
difcerning. And if we grant that in refpect of
its peculiar Atmofphære, the motion of a Co-
met folely confidered as in it, be not fo ftrictly
regular; yet the regular circumgyration of the
Atmofphære, in which the Comet is carryed,
in refpect of the Univerfe, cannot but with
thofe that Cant upon that Hypothefis, enforce
the Confeffion of a great degree of regularity in
the Comets motion: and that Comets, in what
part of the Heavens foever appearing, move in
fome conformity with other celeftial bodyes,
notwithftanding the contrariety or diverfity of
their proper, or peculiar motions from the
                                        reft.

reft , is , beyond all difpute , evident from
their dayly moving, with the reft , from Eaft
to Weft. The manner of making obfervations,
calculating to know the place , the diftance,
fpeed of fuch a Comet , and when it is at near-
eft , or remoteft in its courfe from us , is no
other then in Comets of the fecond fuppofi-
tion. And from this whole difcourfe is mani-
feft ; the Hypothefes , upon which Calcula-
tions are to be made , in order to fuch prædi-
&tions , cannot be particularly in thefe Comets
known before hand, but in feveral fuch Co-
mets may be very different ; and are to be found
out by curious , and diligent Obfervations ;
which , in each refpective Comet , muft be
accurate , and at leaft four in number : and
I think predictions pretended from three ob-
fervations , are over hafty , and, at beft , can
but be conjectural ; nay , in fome cafes , from
four ; as if you be unfatisfyed , I fhall be
ready , by demonftrative inftances , to remove
your doubt. But if of thefe Comets any fhould
be found , to wander , fo as the motion of its
Center cannot poffibly be reduced any thing
nigh unto the fame plane , or regularity in
change of Planes , or that the motion of its
Center , in the fame , or feveral Planes , de-
fcribe not a Line regular , or near unto a re-
gularity , it is not probably eafy to make pre-
dictions at any certainty of truth , of the fu-
ture

ture courfe and motion of fuch Comets. Con-
cerning Comets of the later kind, or Hypo-
thefis; poffibly it is more eafy to anfwer your
Queries in them, then effectually without a
world of time, fagacity and diligence to ac-
complifh and attain the purport of thofe *Anf-*
wers. That the motion of fuch Comets of the
fecond fort, being not truly new; but rather
feldom appearing, and foon difappearing Co-
metical Stars; that, I fay the motion of fuch,
is not without fome regularity, though it may
not be concluded without long obfervations and
frequent experiences of their motion had,
which in fuch feldom appearing, and foon
again difappearing Stars is difficult enough to
attain to; yet if we confider the fteady, un-
biaffed courfe, held by moft, if not all the
reft of the permanent Stars without deviation,
though in different periods and with many dif-
ferent latitudes and declinatiens, and great diver-
fity of motions, it cannot be lefs then migh-
tily probable, nothing in fenfe, reafon, or
experience appearing as yet able to demonftrate
the contrary, but that a fair celeftial regu-
larity may rationally be expected in thofe their
perpetual motions, whether their Centers be
carried in the fame Plane, or in continual
change, and alterations of Planes. And in di-
verfe refpects in fome, if not all of them, as
in moft other Stars, the fame Center may be
<center>G</center> faid

said to move regularly and perpetually in the
same Plane, and also without elenchtical con-
tradiction to move regularly with an incessant
change of Planes: so the motion of any Pla-
net, or other Starr, whose motion we are
versed in, taken concretely and with compli-
cation of that mundane, or general circumvo-
lution, which with all the rest it hath from
East by South to West, though it be regular,
yet it is not without a continual shifting of
the Planets, or Stars center from plane to
plane: whereas the proper motion of the same
Planet, or Starr considered abstractly and se-
vered from the general circumvolution it hath
with the celestial universe, as it is regular, so
it may keep the center of the planet, or Star
constant in the same plane· And the Course,
appearance, and returns of such cometi-
cal Stars will be of much more easy calcu-
lation, if their motions, in the regularity,
propriety and peculiarity of them, be any
wayes reducible, as is abovesaid, unto the
same plane; of which plane by observations is to
be decided, whether it pass thorough the center
of the Æquator and universe; and if the cen-
ter of the universe lye out of it, what is the
perpendicular from the Center unto the plane,
and in the plane, what regular figure, or jour-
ney is described by the motion of the Comets
center, and what are the Diameters or Axis

of

of fuch figures, or centers of fuch motions in
that plane : in the eafieft, and plaineft of all
which cafes, four obfervations are at leaft re-
requifite : but where the regularity of the Co-
mets motion, being irreducible to the fame
plane, is in a continual fhifting from plane to
plane, there many more obfervations may
fcarce be enough, to clear up the regularity of
the Comets motion, fufficiently for eafy cal-
culation : and when a fufficient number of ex-
quifite obfervations have been made, the cal-
culation, as it will hardly be fo accurate, fo
it will fcarce be fo eafy, as in other cafes- The
obfervations, in all cafes and fuppofitions, are
to make difcovery of the exact place of the Co-
mets center at the moment of the obfervations,
Viz. of its longitude, latitude and altitude not
in refpect of Azymuthal arches, but in right
lines drawn between their center and the cen-
ter of the univerfe. And the more exquifite the
obfervations are, they make way for clearer
calculations ; upon which may be founded fo
much the more affured predictions. To an-
fwer therefore your *Q*ueries in relation to
thefe Comets of the fecond fort, or fup-
pofition ; I think it not impoffible, but in
time predictions may, it may be many
Ages beforehand, be made of the Appearing,
or rather Re-appearing of fuch Comets : accor-
ding as, in the regularity of their motion, they

G 2   come

come nearer or are removed Farther off from us. So I remember a person of Quality in the North, with an high measure of Assurance reported to me, that one of the late Comets appearance and Emerging, was by a certain Genœse prædicted at least two years before the Emersion and appearing of it: which if so, is to be referred, I doubt not, to the principle and method, above insisted on. According to which, after different intervals of years, several Comets may appear, one after another, and sometimes several such Comets may appear all at once : and yet there is no clear necessity of the constant re-appearing of the same Comet, in the same Horizon, still at after equal spaces of time. And if the Emerging of such Comets may be prædicted before their Appearance ; upon the same principles, the line of their motion, the casting of their Train, and their future recess, may as well be prædicted, whether before, or after their first appearance: as also when, and where shall be their Accelerations and retardations in motion ; especially, considering the advantage hereunto accruing, from their nearness to, or distance from the Eye of the spectator : not that this makes inequality, or inequability to be, but rather only to appear, in their motion. Upon the same accounts, the dayly rising, setting, and Southing of Comets may be calculated:

lated: as also what parallel, or spiral it describes: and how much it gains dayly in its proper motion : as also its place, as to longitude, latitude, altitude, above any limited. Horizon, at any limite moment : what constellations it shall pass thorough:and in what *Aspect* it shall be posited to any Planet, or other Star: so of conjunctions, oppositions, quadratures, combustions, Eclipses; and many such like affections, in common incident to Comets with other Stars, there may as clear predictions be made : as also when it disappears, not for its remotenesse from us, but as other Stars dayly, because of the Suns propinquity to it, and our view, the times of its not being seen by reason of the circumfusion of the greater light of the Sun, being manifestly calculable ; though of other obscurations by clouds, and such like impediments and interpositions, no artificial prognostick can be made. So in the first Approaches, or last recesses of Comets, by falling into such circumfusions of solar light, the first appearance of the Comet will be later then otherwise, and in the other case, its appearing cease sooner then might be expected : and predictions are accordingly to be ordered. If I have been unusually long beyond the proportion of a Letter, suppose it to be the long Train peculiar to such Cometical, full-lighted, obscure-natured Stars, as you are now Quæ-

ring

ring about. And not to difmiffe you without a
Task ; it were a noble difquifition clearly to
unfold , what it is in thefe Cometical Stars ,
alone, or alfo in their circumfufed Atmofpheres,
that enables the Sun , more efpecially to
affix fuch glorious , long luminous Tayls unto
them more then to other Stars , whether con-
ftant , or new , or rarely appearing Stars ;
which in whatfoever pofition to the Sun , are
ftill feen without any fuch long , beamy , lu-
minous Trains. But I commit this and thofe
other _Queries_ about the falts of the higheft vo-
latiles to your ferious leifure.

The

The sixth Anſwer.

That the motion of the Celeſtial bodies about their particular axes , is not a ſatisfactory proof of their Copernican Syſteme. That Comets though ſuppoſed to be neareſt to the Earth , when they are in oppoſition to the Sun , yet do not thereby prove the Earths motion.

I See that neither the currency of Doctrines, nor the courſe of time , are able to beat off from you your old Philoſophical genius. You can ſwallow nothing upon the mere Tradition of others , nor yet upon their reaſons , till firſt examined, concocted , and digeſted in your own. Your doubts ſeem not cauſeleſs which you move upon that point ; whether the motion of the Celeſtial bodies turning themſelves about , upon their own Axes , may be allowed for a juſt proof of the Copernican ſyſteme. It is a golden and noble branch of intellectual Juſtice , ſo far as is poſſible , mathematically and to a punctillio , to clear and ſet forth the bounders of Truth : yet as your modeſty delivers your judgement , I doubt not , by way of doubt ; ſo I deſire you read my Anſwer , as writ only in the ſame Dialect ; that I , in like manner , may not appear a raſh Judger , of what it ſeems has been concluded by men of great parts and

G 4   learn-

learning. The Artfullnefs of the Inftruments, and diligence of the Obfervers that, from feveral marks in feveral Stars, have noted this their circumvolution, I love and honour: and am apt to conjecture, the like motions may in future be difcovered in other Stars, in which they have not as yet been noted: and fhall be ready to improve the Obfervation in all inferences authorized and recommended unto us by the neceffity of their confequence. But as things, at firft glance, commonly offer a fuller raye then they hold in conftant; fo I fufpect the glimmering probability of the Earths motion according to the Copernican Hypothefis, upon the firft difcovery of this circumgyration of Stars about their own Axes, will, upon farther confideration and deliberation, grow ftill more faint. There being very few Hypothefes, with which (to give it a fhort name) this circumaxal motion of the Stars is expreffely inconfiftent; it not being impoffible, if all things elfe would confift and cohære, but fuch a motion might be accomplifhed in the Hypothefis of Orbes and Sphæres, fo the Star were but unfixed and fet lax and free to move according to its nature in its own orb, by a motion difcharged, different and diftinct from the motion of its orb: So in the old Hypothefis, while they affixed and bound the Starre unto its proper Epicycle, yet in its larger Orb they allowed it to be therein

in carryed fometimes progreffively , fome-
times retrogradely , and fometimes ftationari-
ly , according to the motion of the Epicycle,
or Epifphære in the larger fphære. And with-
out any fenfe as yet , that it deferves a blufh,
I willingly profefs , I want the Lyncean eyes
and judgment , from fuch a circumaxall mo-
ving of Stars , to deduce the Suns, or Earths
central, either reft , or motion : efpecially
the vaft diftance , at which thefe bodyes are di-
ftant one from another , being on all hands
yielded; which if it do not neceffarily , alto-
gether amort, and abolifh the powers , which
one of thefe bodyes may poffibly have upon the
others , yet till fuch their powers over one a-
nother in this point of motion be fairly pro-
ved , it leaves it eafily manifeft , that any of
thefe bodyes may be moved according to its
own peculiar nature , without laying , by fo
long a reach , laws of neceffity upon other Bo-
dyes, at fo great a diftance , for their motion.
As for example the bodyes A. and B. either of
the fame general, or fpecial nature ; they being
fevered and feverally placed at competent di-
ftance one from another ; both, or either may
be at reft , or in motion , neither inferring up-
on the other the analogy either of its reft , or
motion, or to be fufpected fo to doe , till it be
pregnantly either fo obferved , or proved : *Viz.*
That either has upon the other, even at di-
ftance,

ſtance , ſuch a ſtrong and efficacions power·
And if the ſtreſs of the ratiocination hang upon
this; that otherwiſe there would not be a con-
formity between the motions of the Earth , the
Sun , Saturn, Jupiter , &c. to my apprehen-
ſion , nothing is more eaſy, then not only to
turn the edge , but break the back and over-
turn the foundation of ſuch an Argument. For
firſt the motions , or unmovedneſs of theſe bo-
dyes , is not to be made conform unto our re-
tired thoughts , and ſpeculations in a cloſe ſtu-
dy, or to the motions , which we have obſer-
ved, and juſtly concluded certain in other noble
Bodyes-in the world ; but they are rather by all
ingenions Arts and inſtruments , to be each by
its ſelf particularly obſerved , how in Truth
they are , and ſo to be accepted and acknow-
ledged in their appropriate ſciences. And what
greater neceſſity is there of making the Sun , or
the Earth in reſt and motion to be conform to
Saturn, Jupiter , &c. then to be conform to
them in ſhape , magnitude , reſpective diſtan-
ces , which on no hand is aſcerted , but for the
impoſſibility , and demonſtrablenes thereof by
obſervation, rather the contrary ? And why
ſhould there not be as great a neceſſity for them
to be conform in reſt and motion , to the Stars
paſſing generally under the notion and title of
Fixed , which has never yet been offered , be-
cauſe it is clear it cannot be defended ? And
what

what lefs neceffity is there, why they fhould
not conform in the celerity and nimblenefs of
their motion, as well as the form, kind and
manner of it? And what greater necefsity for
conformity of motion in Stars, then in Animals,
in which fome creep, fome fly, fome go upon
feet, fome fwim; neither in their fuppofed and
called Elements can any fuch necefsity of the
fame kind of motion be obferved, though in
the fame Element. Upon the fame way, me-
thod and force of reafoning, when *Jacob* is at
the Dancing, or Fencing Schools in *Oxford*,
Joga muft be Galliarding or Brandifhing it at
the like Schools in *Salamanca*. Every ftone
has not in it the myfteryes of the Magnet, and
feveral Jewels have their feveral fplendours,
virtues and properties: fo feveral Stars confi-
dering their feveral fituations, fhapes, magni-
tudes, diftances and diverfities of nature, can-
not but in reft and motion each have fome-
thing proper and peculiar: at leaft may not
without caufe be fufpected to have otherwife,
till it be pofsible by fome medium to evince the
contrary. And as the Copernican Syfteme re-
ceives not much confirmation from the præce-
dent circumaxal motion of Stars; fo that Co-
mets are neareft the Earth when oppofite to
the Sun; with what greater ftrength doth it,
as others urge, prove the Earths motion? where-
in is requifite that the Hypothefis it felf, be
con-

confirmed by frequent Obfervation in feveral Comets, before it be allowed as general; that as by way of Induction may be inferred, that every Comet when it is neareft to the Earth, is then oppofite to the Sun, and when this is allowed and fairly proved, there is no necefsity by which it follows, or can be inferred from hence, that the Earth hath its annual Revolutions about the Sun: all that is in the allowed, and fuppofedly proved Hypothefis, being many ways accomplifhable, and as eafily and clearly, the Sun moving about the Earth, as the Earth moving about the Sun, in which I doubt not your judgment.

To

The Second
SECTION
of
PHILOSOPHICAL
ANSWERS.

CONTEINING

QUERIES

Relating to the Angle of Contact.

To the right reverend Father in God, *Seth* Lord Bishop of *Salisbury.*

To the right reverend Father in God *John* Lord Bishop of *Chester.*

My Lords.

*A*Lthough *I know, the raw, untrained Reader, upon this last Answer above the rest, will be sure to pronunce his, Q*uis leget hæc? *and to bestow upon it, no other fate then* thuris & scombrorum; *judging all things impertinent, that lye not in the dirty troden way: or cold, which his torpid dullness is unable, at least too slothfull to strike any fire out of; and every thing to be destitute of a kernel, which is too hard for his delicate G*um *to crack the shell of: however, I fear not the rendering my self faulty beyond Apology, in laying this Answer, above all the rest divulged in this present manual, at your Lordships feet. The singular happines of both your Lordships in your studies, drawing in by way of divertisement, what multitudes of others study seriously and purposely for, and and yet miss of, having made veneration to wait upon your jugdment,*

and

and your minds *so teemingly fruitful, in mul-*
titudes of as new as ingenious inventions ; hath
too well acquainted your Lordships , with the
Scufflings which have been in former ages, and
between men of the clearest judgments , and
most excellent parts , about these minute angles
of contact ; being , by general confession , small
beyond all proportion with rectilinearies ; and by
others only denyed to be minute , because they
think they have no quantity at all. Like as in
solar Atomes , when myriades of them are be-
fore us , we cannot be peremptory in asserting a
full sight of any one of them. And however I
may have missed the Corinth I first hoysed sayl
for , I know when retiring from your higher-re-
gion'd speculations you are pleased to make unto
your selves a Platonick divertisement , upon a
feigned perusall of this scrole to a friend , I
doubt not but your Fordships will justify the essay,
and allow that by such sallyes , a firm compre-
hension may at last be attained of the mercurial
niceness of this question , which hath so oft elu-
ded the grasp of those wits , who have formerly
thought they have had it so firm and sure. My
Lords, I know the admirable quality , and migh-
ty merits of the persons that think and have
thought otherwise , then I have delivered my
thoughts here; however I should have dealt faith-
lessly with my friend , if I should have returned
him answer contrary to my judgment. And as I
have

have been morally true to him, ſo what right, or in-
jury I have herein done unto Philoſophical truths
in this nice, and hitherto inexplicable ſubtilty,
none can better decide then your Lordſhips ; at
whoſe feet it is left, by

Your Lordſhips Servant.

W. M.

H   *The*

The Seventh Anſwer.

Concerning angles of contact. That recto-convexe angles of contact are truly angles according to the definition of plane angles. That recto-convexe angles of contact are not deſtitute of quantity. How many wayes one quantity may be greater then another, what kind of homogeneity it is that is requiſite for proportionalls. That angles have figuration as well as quantity. That in angles is obſervable a homogeneity or heterogeneity which is Mathematical, and alſo a homogeneity or heterogeneity which is not Mathematical, nor any way concerning their quantity, but only their figuration, &c. Of ſeveral ſpecial properties of magnitudes and angles. How ſome angles are Conſtituted by the concreſcency and annexion of ſeveral parts and angles which are heterogeneal each to other. Whether like angles in unequal circles, and in loke ſegments of circles be always Equal? That all Equal angles are not like. That all like angles are not Equal.

THough I cannot conſent unto your thoughts about the Angle of Contact, I ſhould be angry with my own raſhneſs, ſhould I be forward to blame your late ſpeculations thereabout; ſeeing ſo many noble and ſprightly wits, and ſuch as have deſerved the honour of *Europe*, ſiding with you. Eſpecially I honour your thorow-converſation in the learned perſons works, whom I perceive you ſo largely

ly to have gleaned after : you could not have plowed in thefe hard grounds with a more worthy Afsiftant. But knowing it is not Complement you expect in Philofophy but Reafon; I fhall in as fhort, as the matter will bear, and as plainly as I can, fumme up the grounds, why I as yet remain fo farre diffatiffyed, that I cannot without injuftice to my own judgment, defert the generous and learned diffenters from your Opinion : or ceafe to averre Angles of Contact to be true quantities , equal ; or unequal one to another , though the higheft multiplex of the greateft citradiametral Angle of Contact, be never able to exhauft , or equalize the leaft right-lined part of the moft acute right-lined Angle.

And that we may not miftake each other about the State of the Controverfy ; and to be fhorter , and more clear in the infuing difcourfe ; we fhall firft fet down the definitions of certain termes, about which I fuppofe there will be no difference between us. As.

1ft. (Our difcourfe extending no further then to plain Angles) that by a Right-lined angle , we underftand a plane angle contained under two concurring fides, or lines, which are two right lines not pofited directly the one in the production of the other, nor parallel the one to the other. As in fig. 1. the Angle *BAC* is a right-lined angle contained under the two

H 2   right

right lines BA and CA; BA and CA not being
fo pofited at A as to make one right line : nor
can they be parallel becaufe of their rectitude
and concurrence.

2^{ly}. That by a crooked-lined *Angle* , we
underftand a plane Angle , contained under
two concurring fides, or lines, which are two
crooked lines ; as in the fig. 2. 3 4. the *Angle*
B*A*C is a crooked lined angle contained under
the two Crooked lines B*A* and C*A*.

3^{ly}. That by a crooked lined angle of con-
caves, or a Concavo-concave angle, we un-
derftand a plane angle contained under two
concurring crooked lines; both which crooked
lines obvert at the *A*ngle firft of all inwards, the
concave fide of their Curvature , towards the
angle under then contained. So in fig. 4. the
angle B*A*C is a crooked-lined angle of Conca-
ves, or a Concavo-concave angle, contained
under the two crooked lines BA and C*A* both
which obvert the Concave fide of their cur-
vature towards the angle under then contained.

4^{ly}. That by a crooked-lined angle of Con-
vexes, or a Convexo-convexe angle, we un-
derftand a plane angle , contained under two
concurring crooked-lines ; both which croo-
ked-lines obvert at the angle firft of all in-
wards , the Convexe fides of their Curvature
towards the angle, under them contained. So
in fig. 2. the angle BAC. is a crooked-lined
angle

angle of convexes, or a Convexo-convexe angle, contained under the two crooked-lines BA and CA, both which obvert the convexe fides of their curvature, firft of all inwards, towards the Angle under them contained.

5*ly.* That by a concavo-convexe angle we underftand a plane angle, contained under two concurring crooked lines; the one of which, at the angle firft of all, obverts the convexe fide of its curvature inwards towards the contained angle, the other the concave fide of its curvature. So in fig. the 3*d.* the angle BAC is a concave-convexe, or a convexo-concave crooked-lined angle, contained under the two crooked lines BA and CA, whereof the one BA obverts the convexe fide of its curvature towards the contained angle, but the other CA obverts the concave fide of its curvature towards the fame contained angle.

6*ly.* That by a mixed lined angle, we underftand a plane angle, contained under two concurring lines; whereof the one is a right line, the other a crooked line. So in fig. 5*th.* 6*th.* the angle BAC is a mixed-lined angle, contained under the right line CA. and the crooked line B*A.*

7*ly.* That by a Recto-concave angle, we underftand a plane angle, contained under two concurring lines; whereof the one is a right line, the other a crooked line, having the con-

cave

cave fide of its curvature obverted towards the contained angle , fo in fig. 5. The angle B.A.C. is drawn a recto-concave angle , being contained under the right line C. A. and the crooked line B. obverting the concave fide of its curvature towards the contained angle.

8 ly. That by a recto-convexe angle , we underftand a plane angle , contained under two concurring lines ; whereof the one is a right line , the other a crooked line , having the convexe fide of its curvature obverted towards the angle contained , fo in fig. *6th*. the angle B A C. is drawn a recto-convexe angle , being contained under the right line C A. and the crooked line B A. obverting the convexe fide of its curvature towards the containd angle.

9 ly. That by an Angle of contact we underftand a plane angle , contained under two concurring lines ; which two lines are neither one and the fame line produced , nor yet in their prefent pofiture and inclination can poffibly cut one another , how farr foever either , or both of them be produced. So in fig. *7th*. and fig. *8th*. the angle BAC. being an angle of contact, if in either figure the crookd-line BA , be produced beyond the point of contact A , unto D, and in fig, *7th*. the crooked -line CA, in figure *8th*. the right line C A. be produced beyond the point of contact *A* unto E; in neither the line CAE can fal into the production of the lineBAD, nor in either
fhall

ſhall the line CAE cut the line BAD. And

10ly. From the figuration, pofiture and pro-
perties of the ſides of ſuch angles of contact; that
by a recto-convexe angle of contact, we under-
ſtand a plane angle being recto-convexe, as
above defined, and being alſo an angle of
contact. So in fig. 8th. if in the recto-convexe
angle BAC, the right line CA touch and cut
not the convexe arch BA the angle BAC
is a recto-convexe angle of contact.

11ly. That, by a recto-concave angle of
contact we underſtand a plane angle of con-
tact, which is alſo recto-concave, as above
defined. As in fig. the 8th. if in the recto-con-
cave angle EAB the right line EA touch and
cut not the concave arch BA, the angle EAB
is a recto-concave angle of contact.

12. That by a concavo-convexe angle of
contact we underſtand a plane angle of
Contact, being alſo concavo-convexe, as
above defined. So in fig. 12th. if RAH being
a concavo-convexe angle, the arch RA touch
and cut not the arch HA then the angle RAH
is a concavo-convexe angle of Contact.

13. That by a Citra-diametral-concavo-con-
vexe angle of Contact, we underſtand a plane
concavo-convexe angle of Contact, both whoſe
conteining Arches lie on the ſame ſide of the
right line tangent, which toucheth both the
arches in the ſame point of Contact, As in fig.

H 4   12th.

12*th*. the concavo-convexe angle of Contact
H*A*R , having both *A*rches H*A* and R*A*
lying on the same side of the right line tangent
B*A*, which toucheth both the *A*rches in the
same point *A*, the concavo-convexe angle of
Contact H*A*R is citradiametral.

14. That by an ultra-diametral concavo-
convexe angle of Contact, we understand a
plane angle of contact being concavo-convexe ,
and having the one arch on the one side of the
diameter passing thorow the Contact point ,
and beyond the right line tangent which tou-
cheth the *A*rches in the same point ; but ha-
ving the other arch on the other side of that
diameter , and on the other side of the right line
tangent. So in fig. 12*th*. the two Circles F*A*H
and *A*DK touching in the point *A*, and AK
being their common diameter and AB being
a right line tangent to both , the concavo-
convexe angle of Contact DAH is ultra-dia-
metral.

15. That by a mixed crooked lined angle ,
we understand a plane angle contain'd under
two concurring crooked lines , of such several
curvatures that it is impossible for them to be
coapted from the angular point the one unto
the other , or the one , or its production to be
coapted to the other , or its production , at and
from the angular point , by any circumduction
whatsoever , so in fig. 9. 10. 11. the angle
BAC

BAC, being a mixed crooked lined angle, AB
as being of a different curvature from AC, can no
way poſſibly be coapted by any circumduction
to AC from the angular point A : and if BA
be produced to D and CA to E the crooked-
line AB, becauſe of its different curvature can
neither be coapted to the crooked line AC
nor to its production AE: nor the crooked
line AC to the crooked line AB, nor to its
production AD, by any circumduction upon
the angular point whatſoever.

16. That by the iſoclitical ſides of an Angle,
we underſtand two ſuch lines, whether right,
or crooked, containing a plane angle, as accor-
ding to their preſent ſite and poſiture, without
inverſion when crooked, moving by the ſides
nearer one to another, will by ſuch motion
at laſt come to be coincident with and coapted
exactly the one unto the other : and that by
an iſoclitical angle we underſtand a plane
angle, contained under ſuch iſoclitical ſides. So
every right lined angle is manifeſtly iſocliti-
cal ; and its ſides are iſoclitical, the ſides by ſuch
motion on the angular point one towards ano-
ther being manifeſtly coincidible and coaptable
So alſo in fig. 3 d. if in the concavo-convexe croo-
ked lined angle BAC. the crooked-lined ſide
BA by moving upon the angular point A
towards the other crooked lined ſide CA, will
at laſt coapt and become coincident with the
<div align="right">crooked</div>

ked lined fide C*A* then are B*A* and C*A* ifocliti-
cal fides and the angle B*A*C anifoclitical angle.

17. That by the anifoclitical fides of an
angle we underftand two fuch lines, whether
the one right & the other crooked, or elfe
both crooked, and whether both of the fame
or different curvatures, containing a plane
angle, fo as according to their prefent fite and
pofiture, without inverfion of either when
crooked, by moving the fides nearer one ano-
ther, by the continuance of fuch motion
they can never be brought to be coincident
with, and coapted exactly the one unto the
other and that by an anifoclitical angle, we
underftand a plane angle contained under fuch
anifoclitical fides. So all crooked lined angles
of concaves, though having fides of like cur-
vature, all crooked lined angles of convexes,
though having fides of like curvature, all conca-
vo-convexe angles, whofe fides are of different
curvatures, all mixed lined angles, whether re-
cto-convexes, or recto-concaves, all mixed croo-
ked lined angles whatfoever, are all of them ma-
nifeftly anifoclitical angles, and their fides ani-
foclitical. So in fig. 4*th*. though AB and AC. be
fuppofed to be of uniform anfwering and equal
curvatures, and likewife in fig. 2*d*. though AB
and AC again fuppofed to be of uniform, an-
fwering and equal curvatures, and in fig. 3*d*. fig,
5,6,9,10, 11. fuppofing *A*B and *A*C not to be of
uni-

uniform, anfwering and equal curvatures, by moving the fides *A*B & *A*C in that their prefent fite & pofiture upon the angular point A, without the inverfion of either, when both are crooked, to bring the fides nearer the one to the other, it is manifeft that by the continuance of fuch motion they can never be brought to a coincidence with, and to be coapted exactly the one unto the other, and fuch alfo are all angles of Contact whatfoever, as in fig. 7, 8, 11. *A*B & *A*C are as uncoincidable and uncoaptable as in the former, except only ultradiametral, concavo-convexe angles of Contact, which are of equal and anfwering curvatures.

18. That by an Angle of curvature, or coincidence we underftand a plane angle, contained by any two parts of a crooked line at the point of their concurrences any where to be imagined, or taken in crooked-lines. So in the 7*th*. fig. let BAD be the circumference of a Circle, Parabola, Hyperbola, Ellipfis &c. At the mean point A, the fides BA and *A*D contain an *A*ngle of Curvature, or coincidence.

19. That by autoclitical curvature, and fo by an autoclitical crooked line, we underftand fuch a crooked line, as paffing from the angular point of a right lined angle, between the two fides, by the inclination of its curva-
ture,

ture, keepeth the convexe side of its curvature,
conftantly obverted to one of the sides, and
the concave side of its curvature conftantly
obverted to the other. *As* in fig. 17. in the
right lined angle BAC, the crooked line *A*EF
keeping its convexity conftantly obverted to
the side AB, and its concavity to the side AC,
or so much of it as is intercepted between the
interfection at F and the angular point at *A*
the crooked line *A*EF is autoclitical, *i. e.* the
convexity is all on one side, and the conca-
vity all on the other.

20. That by antanaclitical curvature and so
by an antanaclitical curve line, we underftand
fuch a crooked line, as paffing from the an-
gular point of a right-lined angle, between
the two fides, by the inclination of its curva-
ture, hath the convexe part of its curvature,
fometimes towards the one side of the right
lined angle, and fometimes towards the other
fide of the right lined angle, *i. e.* the con-
vexenefs and the concavenefs are not con-
ftantly on the fame feveral fides. *As* in fig. 17.
in the right lined angle BAC, the crooked line
AGH obverting the convexity at G towards
AB, and at H towards AC is antanacliti-
cal.

These definitions præmifed, to give now
the true State of the controverfy; let there be,
as in fig. 12. two equall Circles AHH and ADK

touch-

touching in the point A : and let AG be the
Semidiameter of the circle ADK : and AB
be a right line tangent touching both the Cir-
cles in the point A : and let *A*EL be a greater
circle then either , touching both the former,
and alſo the right line tangent in the point A :
and from the point A draw the right line AC
at pleaſure, cutting the circle *A*DK in the point
D, and the circle AEL in the point E : now
therefore whereas you ſay that the Recto-con-
vexe angles of Contact BAE and B*A*F are not
unequal , and that neither of them is quanti-
tative ; and that the crooked-lines E*A*, DA,
HA are coincident, *Sc.* ſo as to make no *An*-
gle with the right line *A*B , or one with ano-
ther ; and that the right-lined right angle B*A*G
is equal to the mixt-lined , *i. e.* Recto-concave
angles of the Semicircles E*A*G and D*A*G ſe-
verally, and that thoſe angles of the Semicir-
cles E*A*G and D*A*G are equal the one unto
the other ; and that the mixed lined Recto-con-
vexe angle F*A*G is ſeverally equal to all , or
any of the former ; and that there is no hete-
rogeneity amongſt plane angles , but that they
are all of them of the ſame ſort, and homoge-
neal and undevideable into parts ſpecifically
different , diſtinct and heterogeneal in reſpect
of one another, and the whole ; and that to
any mixt , or crooked lined angle whatſoever
that is quantitative it is not impoſſible to give
an

an equal right lined angle : I acknowledge
for all thefe things you have difputed very fub-
tilly ; yet I muft with a clear and free judgment
own and declare a diffent from you in them all,
for the reafons to be alleadged in the infuing
Difcourfe.

To clear all which , nothing can be of
higher confequence in this Queftion, then truly
to underftand the nature of an Angle , what an
Angle is , what is an Angle , and what is not.
And to exclude the confideration of Angles
herein unconcerned ; they are plane angles
i. e. fuch as are contained by lines , which lye
wholly , and both in the fame plane , the dif-
quifition of whofe nature we are now about.
And fuch is the affinity , which the inclination
of one line hath to another in the fame plane,
with the nature of an Angle , that without it
a plane angle cannot be defined , or conceived.
For where there is rectitude and voidnefs of in-
clination , as in a right line and its production,
there never was juftly fufpected to be any thing
latent of an angular nature ; nor between pa-
rallels, for want of mutual inclination. But yet
the inclination of line upon line in the fame
plane , is not fufficient to make up the nature
of an Angle : for in the fame plane one line
may have inclination to another , and yet they
never meet , nor have in their infinite regular
production any poffibility of ever meeting ; as
the

the circumference of an Ellipſis, or circle to a right line lying wholly without them, without either Section, or Contact; or the Aſymptotes in conjugate Sections, which though ever making a cloſer appropinquation to the circumferences of the conjugate figures, yet infinitely produced, never attain a concurrency. And though by poſſibility the inclined lines might meet, yet if they do not, an Angle is not conſtituted : only there is poſſibility of an Angle when being produced they ſhall meet. So as to make up the nature of an Angle two lines muſt be inclined one to another, and alſo concurre, or meet to contain on their parts a certain ſpace, or part of the plane between their productions from their point of concurrence, or their angular point. And this being the general, and proper nature of a plane angle; as it is manifeſt, that in a right line there is no Angle; ſo it is dubious what is to be judged of thoſe curve, or crooked lines, whoſe curvature is either equall, uniform, or regular; &c. whether in ſuch lines there be not at every mean point an angularneſs, the lines ſtill lying in the ſame poſiture. It is clear in a right line no mean point can be taken, at which the parts of the right line in their preſent poſition can be ſaid to have any inclination one to another : that though the parts concurre, yet they want inclination : but in the circumferences of circles,

cles, Ellipfes, Hyperbola's, Parabola's and
fuch like curve, or crooked lines, being of
equal uniform, or regular curvature, no point
can be affigned at which the parts in their pre-
fent pofition have not a fpecial inclination
one to another. So as, as before, inclination
of lines and concurrency making up the nature
of an angle; it feems not reafonable to deny
angularity at any point of fuch crooked lines,
however their curvature be either uniform,
equal, or regular. But I know it will be faid
that fuch crooked lines of equal, uniform &
regular curvature are but one line, and there-
fore, by the definition of an angle, cannot
contain an angle; which requires two lines to
its conftitution. To which I anfwer, that in
like manner, two concurring right lines may
be taken for one line continued though not
in its rectitude, and then the confideration of
angularity between them is excluded : however
by reafon of the inflexion and inclination at
the point of incurvation, there is an aptitude
in that one produced line, there to fall, and
diftinguifh it felf into two, with inclination of
one unto the other, and fo to offer the con-
ftitutive nature of an angle : and fo it is at
every point of fuch crooked lines, whofe cur-
vature is equal, or uniform and regular; being
one, or more lines, according as by our con-
ceptions they are continued, or diftinguifhed

And

And as a continued rectitude , such as is in
right lines , is most inconfiftent with the na-
ture of an angle , fo what fhould be judged
more accepting of the nature of angularity ,
then curvature is , being thereunto contra-
rioufly oppofite , and by all confeft moft what
to be fo , faving in the aforefaid cafes when
curvature is equal , uniform , or regular. But
why fhonld equality , uniformity , or regula-
rity of curvatures , be fo urged in the concern
of angularity , by none of which is angularity
either promoted , or hindered ? For there
cannot be greater equality , uniformity , or
regularity , then is to be found in Rectitude as
well as angles , in right lines as well as Circles;
the things conftitutive of the nature of an
angle , being things quite different from them ,
Viz. inflexion or inclination and concurrence;
which are indifferently found in all curva-
tures equal and unequal , uniform , or not uni-
form , regular , or irregular : and befides are
infeparable from them : curvature and a con-
current inflexion , or angularity being but as
two notions of the fame thing , confidered
as under feveral refpects : *Viz.* curvature is the
affection of a line confidered as one : the fame
being angularity and inclination , when from
any point of the curvature the fame line is
confidered as two : fo two concurrent right
lines are faid to be one crooked line : but

I   when

when a right-lined angle is said to be contained
by them, they are then considered as two from
the point of their incurvation, or inclination:
and equality, and uniformity, and regularity
of curvature implying equality, uniformity
and regularity of inflexion, or inclination,
it is so far from concluding against angula-
rity, that it inferrs it with an additament,
viz. of equal angularity, uniform angularity,
and regular angularity; as might be at large
declared in the special properties of several
crooked lined figures. And in several uniform,
and regular crooked lined figures, there are
some special points, offering even to view and
sense a clear specimen of a more then ordinary
angularity, without any such loud calling
for the strong operations of the mind; as
the vertical points in conjugate figures, in
parabola's, and the extream points of either
axes, in ellipses, and the like, And in the
definition of a plane angle, all the inclina-
tion, which is required in the concurring sides,
is only, that their inclination be μὴ ἐπ᾽ εὐθείας
i. e. so as the sides make not one and the same
right line, as it is generally understood: howe-
ver so far are we from imposing upon any
against their judgements, this special sort of
angles, that we readily acknowledge, if the
words μὴ ἐπ᾽ εὐθείας, in the definition of a plane
angle, be forced from the commonly received
                                                     sense

fenfe, of the fides not lying in the fame right-line, to fignify the non-coincidency of the production of the one fide with the other, then according to that glofs upon the definition of plane angles, this whole fort of angles is to be rejected; wherein every one is freely left to his own judgement; the difference being a queftion and quarrel about words, more then matter, and not concerning the prefent controverfy.

However from the whole it out of Controverfy appears, that recto-concave angles of Contact are true angles, contained under two inclined fides, concurring : alfo that ultradiametral concavo-convexe angles of Contact, whether lefs equal, or greater then two right right-lined angles are true angles, for the fides are inclined and concurr : of their concurrence can be no doubt : and that they are inclined muft of neceffity be yielded, feeing they neither lye in rectitude, nor which to fome might be a caufelefs fcruple, the one in the production of the other : fo as their angularity is clear beyond all doubt : the inclination underftood in the definition of an angle being generally any pofiture of one line with regard to another in the fame plane, fo as both neither fall in one right line, nor be fituated parallel one to another, nor under fuch a manner of oblique extenfion, as may

ren-

render their concurrence impoffible : it is not
only the oblique pofiture of one line in refe-
rence to another , as contradiftinguifhed from
perpendicularnefs in acute and obfufe incli-
nations : but fuch is the comprehenfivenefs of
its fenfe, in the prefent acceptation , that every
perpendicularnefs it felf is taken for an inclina-
tion.

So likewife it cannot reafonably be denyed
but thofe fpecial angles of Contact , which
are the chief fubject of this prefent con-
troverfy , I mean recto-convexe angles of Con-
tact , are truly angles ; except either the in-
clination of their fides , or their concurrence
could be called in queftion ; nothing elfe being
requifite unto the nature , or comprized in the
definition of an angle : and the like is to be
judged of all other concavo-convexe and con-
vexo-convexe angles of Contact. But you faid
that in the recto-convexe angle of Contact
the right-line tangent and circumference make
no angle , becaufe the tangent is not incli-
ned to the circumference , but coincident with
it. What myftery of reafon , or force of *Argu-*
ment fhould be in this deduction ? if you fay
it is coincidence , you mean by non-inclination;
I readily yield , where two lines become coin-
cident , their former angle is thereby extinct ;
as thereby they come under the confideration
but of one line : as when two right-lines , or
two

two ifoclitical crooked lines, are, one of each
of them, fo moved about the angular point,
till the two lines become one. But where is any
fuch coincidency between right-line tangents
and circumferences ? or what poffibility is
there of any fuch coincidency : a crooked
line and a right-line may no doubt be com-
menfurate, or proportionable in length, but
in pofition it is impoffible : and if we ima-
gine the tangent bowed to fuch a coincidence,
then it is not any longer a tangent, or
right-line, but a circumference. And though
as you urge, a right-line circumduced about
any middle point in the fide of a regular po-
lygone, at laft becomes coincident with the
fide, and loofes all inclination and angulari-
ty with the fide; what doth that concern, or
how doth that prove the non inclination of
the tangent to the circumference ; your felves
fometimes in every point, fave only in the
point of Contact acknowledging an inclina-
tion; and as is elfe where hinted, in the point
of Contact alone, and abftractedly, the incli-
nation it felf, which is the habitude of the
concurring inclined fides, is not to be fought,
but only the particular termination of their
inclination there. But you will fay, it is
non-fecancy which is meant by this coinci-
dency : and what I pray is that more then
the Lateran bells to the concern, or confti-

tu-

tution of angles ? Are there not many regular curve lines produced , some infinitely without Section , in which especially the circumferences of circles, you are pleased sometimes every where to think you have cause to imagine an angle. It were meet to know the meaning of such odly connext terms , before reasonings upon them be regarded in questions of weight.

That the Contact of the angular sides , is as different from coincidency , as from secancy , is most unquestionably apparent in angles of such sides , as are capable of all three , *Viz.* Contact, secancy , and coincidence. For example in fig. 16. let there be two Arches of equal Circles DAF and BAC. touching one another in the point A. If the arch DAF be circumduced about the point A as an unmoveable center; at after an infinite succession of secancies , at last all will terminate in a manifest coincidency , and the arch AF be coincident with the arch *A*B , and the arch AD be coincident with the arch AC. So as secancy , Contact, and coincidency are distinguishable one from another with as much ease and clearenefs , as an odde number from an even. But if it be urged, that you affert not a coincidency between the arch. D*A*F and the arch BAC , but only that GH being a right line and touching the arch B*A*C in the point *A* , that I say the right line

GH

GH and the arch BAC are coincident. The
vanity of this may be evinced ; in that by the
fame reafon it followes , that the right line
GH muft be coincident with the arch DAF,
and fo the arch DAF coincident with the arch
BAC ; the contrary of which is above-fhewn
and confeffed : and befides hereupon
fhould two arches BAC and DAF being con-
vexo-convexely pofited , and the right line tan-
gent GH be all coincident ; which I leave for
others to fay rather then my felf.

When a right line tangent and many crooked
lines of different curvatures, all touch together
in one and the fame point, as in fig. 13 ; you
fay , though without the angular point of Con-
tact the fides are varioufly divaricated one from
another , yet in the point of Contact , they have
not feveral inclinations , for you fay they have
no inclination at all. The truth is the angular
point of its felf is not capable of inclinations,
nor for the indivifibility of its nature can by
any poffibility comprehend them : yet that lines
concurring in one only point , and prefently
after receding each from the other , fhould not
be inclined each to other in or at the point of
their concurrency , whether it be by Contact,
or Section , though in the cafe of Contacts the
inclination be lefs, then can be expreffed by
the inclination of any right-line upon a right-
line, is abfolutely unconceiveable, there being

I 4   no

no lineary coincidence, but only of one point
between them. For as elfe where, in the fame
plane, neither of point to point, nor of point
to line, nor of point to plane can be any in-
clination : but in the prefent cafe of plane an-
gles, inclination muft be of lines, and may be
of them in the very point of concurrence ; or
elfe from the point of concurrence they would
not part feveral wayes: for it is their diverfe
inclination at that point, which makes their
departure one from another when they depart
from thence. And as even in the angular point
of right-lined angles the lines have the fame in-
clinations as elfe where , fo in all other angles,
fave only fuch angles of Contact , as are lefs
then the leaft right-lined angles , by drawing
right-lined tangents to the arch , or arches at
the angular point , is fhewn in right-lines,
either the very inclination of the fides , in , or
rather at the angular point, or elfe the leaft
right-lined inclination , which is greater , or
the greateft right-lined inclination , which is
lefs : for though they may differ much in their
diftance and divarication one from another,
without the point of their concurrence ; in the
point of their concurrence , without much
abfurdity , they may be faid to be equally di-
ftant, _i. e._ not at all diftant there one from
another , for there they are not indeed at all
diftant any of them from the reft : yet it doth
not

not hence any way follow , that in like man-
ner it may be said of them without abfurdity ,
that in the point of concurrence they have
equal inclinations , *i. e.* no inclinations one
to another: for though in the point of con-
currence it is truth , they have no diftance ,
yet it no wayes as may appear to thofe that will
confider equally follows , that they fhould
there have no inclination.

Befides , that the urging of the coincidency
of the fides in recto-convexe angles of Con-
tact is moft directly oppofite to the nature and
properties of the fpecial lines under which fuch
angles are contained. For , tis the fpecial
propertie of fome lines that they can touch ,
but they can no way be coincident ; as Arches
of unequal and unlike curvature : and a right-
line and a crooked line : fome can be coin-.
cident , and can no way touch , as right lines :
fome can both ; as Arches of equal and an-
fwering curvature , which fet concavo-con-
vexely , and citradiametrally cannot touch ,
but will fall into a coincidence : but pofited
convexo-convexely , cannot be coincident , but
may conftruct an angle of Contact.

So as the chimæra of the coincidency of the
fides in recto-convexe angles of Contact , if
perfifted in , is worth laughing at , and like
his Philofophy who when every one was at
his high Lavolta's , denyed the poffibility of
                                    mo-

motion in the world : But to juftify the non-
inclination of the fides againft the eye and
reafon , this horridly diftorted Monfter of
their coincidency was introduced. Indeed if
they be coincident they make no angle : But
it will cramp the underftanding of an *Oedi-
pus* to declare how either a right-line , or a
crooked-line , touching another crooked-
line in one only point and no more , fhould
ever be conceived , notwithftanding to be
coincident with the production of the
other crooked-line , whether the tangen-
cie of the crooked-lines be concavo-con-
vexe , or convexo-convexe , *i. e.* the one
within the other , or elfe the one turned away
from the other. But you'll fay , you affert
coincidency only in the point of Contact.
I anfwer , that's frivolous , not to fay ridicu-
lous and impertinent ; for coincidency in the
prefent queftion of angles is taken as oppofed
to inclination , which is an affection and pro-
priety of the concurring fides of the angle ,
not only of the angular point taken by it felf
abftractedly. As inclination cannot be in a
point , though it may be at a point , fo a point
cannot be faid to be inclined unto a line , efpe-
cially it felf being in the fame line : it may
be faid to hold fuch and fuch a diftance from
the line , when it is without it , but not to be
inclined unto it. And if the being of the point
of

of Contact , generally as a point of the one fide , in the tangent line , as in the other fide , whether right , or crooked , make a coincidency deftroying inclination ; then all inclinations and angles whatfoever are deftroy'd ; and every where will be a coindicency, for that is common to every angle to have the angular point ftill common to both fides : and the fecant angles might as well be faid coincident , as the tangent angles, For what you fay , that it is tangency as oppofed to fecancy, that you mean by coincidency ; I anfwer the gloffe is improper and befides the anvil and tangency undenied , but in this cafe impertinently by you alleadged , till it be proved that tangency in one only point , and no more , doth quite annul and deftroy the inclination of the lines , though on both fides of the Contact clearly receding the one from the other : till which be done happinefs to my friend and no longer.

I might adde that it is Touching which is only mentioned in the definition of plane angles ; but I fhall difpatch all by fetting the cafe before you in this Diagramme in fig. 12. if the right line BA touch the arch LA in the point A and the right-line LB be fo drawn as to touch the arch LA in the point L, here now is a plane on every fide bounded by two right-lines , *Viz.* BL and BA and one crooked

ed-line , *Viz.* the arch LA. That LBA is, an angle will not be doubted : and becaufe the three lines perfectly bound in and limit a plane on all fides ; the Arch LA can neither be coincident with the right line LB , nor with the right-line AB , and in the angle LAB , the lines LA and BA ; and in the angle ALB , the lines LA and BL concurring without coincidency , and without lying in the fame right-line , or the one , fo much as , in the production of the other ; inclination , and fo the true nature of an angle , cannot be denyed them ; according to the moft fevere limitations and hardeft gloffes , that can with any reafon be deduced from the definition of a plane angle. And to make all clear let a Paper , or other plane be cut in the form of the mixed-lined Triangle ALB and the wildnefs of queftioning the angularity of the two recto-convexe angles of contact LAB and BLA will be clear to all perfons both rude and learn-ed. I take it therefore for granted ; that , all fufpicion of coincidency and non-inclination , in whatfoever pertinent fenfe , of Contact-angle fides , being evicted and fent of the fcene, all recto-convexe angles of Contact are truly angles.

To paffe now unto another of your Thefis's in which you peremptorily conclude Recto-convexe angles of Contact to be devoyd of all
quan-

quantitativenefs ; when I urge their quantita-
tivenefs, I mean not that they can at plea-
fure be devided into parts in any given and
limited proportion, or by a mathematical ho-
mogeneity holding any proportion with the
Angle of Contact devided ; only that from the
angular point between the fides of the leaft
Recto-convexe angle of Contact infinite other
lines may be drawn, dividing the angle, though
heterogeneally. And certainly its being an an-
gle of Contact cannot in the leaft prejudice its
quantitativenefs : for it is moft apparent that a
convexo-convexe angle of Contact contained
under Arches of equal curvature, may be di-
vided into two mixed-lined, i. e. recto-con-
vexe angles of Contact, which hold propor-
tion of equality one with another, and each
of them is in fubduple proportion to the con-
vexo-convexe angle of Contact, which was
divided : and infinite numbers of Contact-an-
gles of feveral forts may be adjoyned one unto
another, diftinct in their fituation, without
drowning and extinguifhing one another and
each lying without the other : which is not
true of indivifibles when they are adjoyned
to, and touch one another. Befides the recto-
concave angle of Contact is greater then the
greateft right-lined angle, at the angular point
of Contact having a manifeft inclination of
the fides; for concurring, they neither are one
right-

right-line , nor one crooked-line. Moreover
ultra-diametral concavo-convexe Angles of
Contact may be equal to two right right-lined
angles , or greater fometimes : As in fig. 12.
if the two Circles FAH and ADK be equal
and touch in the point A , and AK be dia-
meter : and AR be another Arch falling
beyond the diameter AK and beyond the right
line tangent AB : it is manifeft that the con-
cavo-convexe ultra-diametral angle of Contact
DAH is as to the receffe of the fides equal
to two right-lined angles : for the recto-con-
cave angle of Contact BAH is equal to two
right right-lined angles , deducting the recto-
convexe angle of Contact HAS : and the recto-
convexe angle of Contact BAD is equal to
the recto-convexe angle of contact HAS ;
therefore the ultradiametral concavo-convexe .
angle of Contact DAH is equal to two right
right-lined. angles : and therefore the ultra-
diametral concavo-convexe angle of Contact
DAR is greater then two right right-lined
angles.

And then , what greater Monfter is difco-
verable in the doctrine of the quantitativenefs
of the recto-convexe angle of Contact ? it is
demonftrated that between the right-line tan-
gent and the Arch which it toucheth , no
right-line can paffe : i. e. the recto-convexe
angle of Contact is lefs then the leaft right-
                                                                lined

lined angle : but why fhould hence be inferred
that the recto-convexe angle of Contact hath
no true quantity ? you will fay , becaufe a right-
lined angle is infinitely divifible into lefs and
lefs parts , and therefore muft at laft be lefs then
the recto-convexe angle of Contact , if the
recto-convexe angle of Contact be a true an-
gle , having truth of quantity. I anfwer , after
all poffible divifions of a right-lined angle into
parts of its owne kind , I mean fuch as are made
by right lines , the leaft part is ftill a right-lined
angle ; then which the recto-convexe angle of
Contact is moft fairly demonftrated to be ever
lefs : *i. e.* the inclination of its fides is ever lefs :
and the convexe arch will ftill at the angle fall
within : fo as truly from thence may be inferred,
that the recto-convexe angle of Contact can ne-
ver be either a right-lined angle , or equal to it,
or greater than a right-lin'd angle:but that there-
fore it is no angle , and hath no true quantity at
all, becaufe it hath not the quantity of a right-
lined angle , is a wild and perverfe inference,
and elfe-where difproved.

And why fhould the quantitativenefs of the
recto-convexe angle of Contact be called in
queftion , becaufe it is demonftrated lefs then
the leaft right-lined angle ? If between the
right-lined tangent and the arch a right-line
could be drawn , then you would confeffe the
quantitativenefs of it as undeniable : and why
doth

doth not the paffing of a thoufand crooked-
lines from the angular point , between the fides
of the recto-convexe angle of Contact , as well
prove its quantity and divifibility , as the paf-
fing of one right-line between them could :
there being equal force of proof from the one,
as from the other ? *And* when a right-line
& two equal Circles , all three touch in the fame
point ; there are two equal recto-convexe
angles of Contact adjacent the one continuedly
to the other , and fituate the one without the
other : which in indivifibles is impoffible. *And*
when the queftion is of the quantity of angles ,
what is it we enquire , but only , what is the
inclination of the fides , efpecially at the angu-
lar point ? *And* in recto-convexe angles of Con-
tact , the anfwer is ; there is no inclination at
all as of a right-line to a right-line , but only
as of a crooked-line to a right-line : that it
were wildnefs to fay ; becaufe in recto-convexe
angles of Contact , there cannot be the incli-
nation of a right-line to a right-line , that
therefore the fides meeting and parting one
from another , and not lying both in a right-
line , do make no inclination one to ano-
ther.

And feeing the convexo-convexe angle of
Contact contained under two Arches of equal
curvature is dividable into two equal parts by
a common right-line tangent to them both ;
the

the quantitativenes both of the convexo-con-
vexe angle of Contact , and of both the recto-
convexe angles of Contact,clearly appears;
though the quantitativenes of every one of them
be demonſtrable to be leſs then the quantity of
any the leaſt right-lined angle whatſoever : be-
cauſe right-lines cannot contain a leſs angle
then is agreable to the inclinations they poſſi-
bly can have one to another : whereas in crook-
ed lines generally , theſe are the conſtant pro-
perties of curvature ; though the angles of coin-
cidence, or curvature,as we for method, brevity
and diſtinction ſake named them,may be very
unequal one to another , according to the de-
grees of the inflexion , inclination and curvatu-
re; yet conſtantly the angle on the concave ſide
from any point of the curvature manifeſtly if it
be equal , uniform and regular , is ſtill greater
then any the greateſt acute , right , or obtuſe
right-lined angle : and the angle between the
convexe arch and a right-line tangent , at the
ſame point , is conſtantly leſs then any right-
lined angle : and yet either may be made ſtill
infinitely leſs , or greater , the recto-convexe
angle of Contact ſtill remaining leſs then the
leaſt right-lined angle , and the angle of curva-
ture , or coincidence greater ſtill then the
greateſt right-lined angle : which is as much as
to ſay, that in curvatures the difference between
the angle of curvature and two right right-lined
<center>K</center>   angles,

angles, cannot be a right-lined angle; as in truth it cannot, nor is in reafon fo to be expected, but of neceffity it muft be a mixed-lined, *Se.* a recto-convexe angle : and is the recto-convexe angle of *C*ontact at the fame point. And that a recto-convexe angle of *C*ontact, by no multiplicity can equal, or exceed a right-lined angle, doth not difprove either its angular nature, or its quantitativenefs, both which are otherwife cleared, but it is rather a confirmation of the heterogeneal difference, which is between the angles of the one fort and the other. And in my judgement there needed no greater argument of the quantitativenefs of recto-convexe angles of *C*ontact, then the abfurdity following upon the contrary doctrine; that the angle of a femi-*C*ircle, and a right right-lined angle are equal, *Viz.* The whole and a part, the one being a mixed-lined and the other a right-lined angle; and in the indeavour of coaptation, and being coapted on one fide, the other fide all the way falls within, or without the other, fo as both the fides of the one angle are impoffible to be coapted to both the fides of the other, nor will both lye within both the fides of the other. And the angles of femi-*C*ircles muft either be confeffed unequal in unequal *C*ircles, or the curvature of unequal circumferences, be manifeftly againftthe truth afferted to be equal. And if ftill they be

aver-

averred to be equal , its defired their equality
fhould be demonftrated , and the way of ad-
meafuring their equality fhown.

But you will fay , if recto-convexe angles
of Contact be quantitative , why can they not
exhauft any other angle whatfoever contained
under the very fame fides; for poffibly you will
urge , that we fhould not queftion the homo-
geneity between fuch angles ? To this I anfwer,
without examining what homogeneity may in
other refpects be between them ; that a quan-
titative and mathematical homogeneity can
with no reafon between them be imagined :
becaufe the difference which is between them
is a right-lined angle , to which all angles of
Contact whatfoever are heterogeneal ; and
your felf will not affert any mathematical
homogeneity , or at leaft proportionablenes
which as to this purpofe is all one between any
recto-convexe angle of Contact and a right-
lined angle. As in fig. 18. let *K*AB be a recto-
convexe angle of Contact , and *K*AF another
recto-convexe angle under the very fame fides :
and let *A*D be a right-line tangent upon the
arch AF : therefore the recto-convexe angles of
Contact D*A*F and *K*AB being equall , the
right-lined angle *K*AD is the difference of the
recto-convexe angle of Contact *K*AB and the
other recto-convexe angle *K*AF under the di-
varication of the very fame fides. So as it is

K 2   im-

impoffible to divide this, or any other angle whatfoever, which is not ifoclitical, to divide, I fay, all of it into any numbers at pleafure given of parts, which fhall be homogeneal all of them one unto another: for how many foever be homogeneal, the angle of Contact, or that which is taken out of it, or that unto which it adhæres, will have and make heterogeneity.

And if you fay, how can recto-convexe angles of Contact be faid to be parts of fuch concrete and compofite angles, if unable by any multiplicity to exhauft the compofite angle? omitting the anfwer that parts are fometimes effential and of the definition, and yet by no multiplicity can equalize the whole; as four angles in the definition of a Tetragone, and a foot line in the definition of a foot Cube: I an-fwer chiefly, that where the integral is hete-rogeneal as here, and made up, and properly and naturally refoluble into feveral heteroge-neal parts, and connot be divided into any num-bers at pleafure of parts all homogeneal, there fome parts may never be able by any multipli-city to equalize the whole, or fome other he-terogeneal parts. And elfs-where that under the fame coaptable fides may be angles different in their ultimate kind in fome farther refpects, though not without proportionablenefs in this inftance, is moft apparent among right-lined angles, by comparing a right right-lined an-
gle,

gle, and an acute right-lined angle, and an obtufe right-lined angle together; which all receive their fpecifick differences from the fpecifick differences of their inclinations : in right right-lined angles the inclination being no more one way then the other , no more from the angle fide , then to the angle fide , *Sc.* perpendicular : in acute angles the inclination being to the angle fide : and in obtufe angles the inclination being more efpecially from the angle fide.

*A*nd yet though we defend the quantitativenefs of recto-convexe angles of contact we are equally obliged to affert their improportionablenefs to right-lined angles : nor will there be any difficulty in anfwering that fuggeffion you caft, though in anothers name ; that a recto-convexe angle of Contact is in proportion lefs then a right lined angle , as being both homogeneal ; and that by the multiplicity of the recto-convexe angle of Contact , an angle may be made equal to a right lined angle , or greater , only by changing its kind , *Sc.* into a right-lined angle : *Viz.* in the fame manner , as an acute right-lined angle , being lefs then a right , or any obtufe right-lined angle , by its multiplicity may with change of its fort and kind become equal , or greater then a right , or any given obtufe right-lined angle. To which may be anfwered , omitting what kind of diftin-

ction

&ion it is which is between acute , right and
obtuſe right-lined angles , as not pertinent to
the preſent controverſy ; it would be well done
to ſhew what multiplex of any recto-convexe
angle of Contact , is equal to what right-lined
angle , that ſo a right-lined angle might, upon
that proportion , be formed equal to , or leſs
then the recto-convexe angle oi Contact ; con-
trary to what has been clearly demonſtrated,
and is generally by all conſented to in Geo-
metry. Though acute and right right-lined an-
gles are leſs then any obtuſe whatſoever that
are right-lined ; however they hold propor-
tion one with another : but recto-convexe an-
gles of Contact cannot be demonſtrated to hold
any proportion with any right-lined angle, but
clearly the contrary. And as by the divari-
cations of the ſides of an acute right-lined an-
gle is made a Geneſis of a right and infinite ob-
tuſe right-lined angle : there cannot ſo by the
divarication of the ſides of a recto-convexe an-
gle of Contact be effected any Geneſis of right-
lined angles , but only of recto-convexe mixt-
lined angles, whoſe ſides concurre by way of ſe-
ction;and between which and the recto-convexe
angle of Contact is no proportion, as there is be-
tween the divaricated acute right-lined angle
and the other right-lined angles created from
that divaricacation. The reaſon of which is clear,
for that the recto-convexe angle of **Contact** be-
<div align="right">ing</div>

ing demonſtrated of it ſelf to be leſs then any
right-lined angle whatſoever, by the divarication
of the ſides of it, there are continually greater
and greater right-lined angles added to it, ſo
creating an improportionality between the one,
and the other.

The compariſon made between Cyphers and
angles of Contact, to draw the one, as well
as the other into the notion of nullities, is un-
happy enough : for ſingle and ſeveral Cyphers
are not greater, or leſs in power one then
another (though they may make other fi-
gures to be ſo) as the angles of Contact are,
and may be made larger, or leſſer, and many
of them one without or within another, con-
tiguouſly and continuouſly conjoyned together,
with enlargement, or diminution of their an-
gularity; which is impoſſible in indiviſibles,
and unappliable to, and unintelligible of, mere
nullities.

That your Lemma is without exception,
and without proof might have been admit-
ted : *Viz.* That two quantities by the ordi-
nate application, or motion of a line, or
plane, increaſing, or decreaſing proportiona-
bly, whether by a proportionality in the ſame,
or different powers, when the ordinate ap-
plication, or motion attains the end, and
bounding term of the one quantity, it at the
ſame moment reacheth & hath attained the

K 4   boun-

bounding term of the other , and when it hath paffed the one it hath paffed the other. But the objection againft the quantitativenefs of the recto-convexe angle of Contact, which you would hereupon found , hath no reafon to expect the like allowance. You fay , the right-lined right angle at the point of Contact contained under the right-line tangent, and the diameter of a Circle , equally with the circumference intercepted , increafeth , or decreafeth by the motion of the diameter upon the point of Contact, as a Center; which is true and acknowledged : and whereas you fay , that therefore when the diameter leaves nothing at all of the circumference in its circumvolution about , un-run over , but attains the laft bounder and termination of it , in the point of Contact, then as the circumference is quite exhaufted and vanifhed , fo is the angle too : this alfo we acknowledge to be undeniable. But whereas when the diameter is come fo near the right-line tangent in its circumvolution upon the angular point of Contact , as to intercept nothing at all of the circumference between them , you then imagine ftill an angle remaining , which you fay , is either the recto-convexe angle of Contact , or not lefs then it ; you herein forget the force of your own manifeftly true lemma , which you took fo much pains to prove ; except againft Geometrical ,

cal demonſtration you could diſcover a poſ-
ſibility of dividing a recto-convexe angle of
Contact by a right-line: for it is out of doubt
and in Geometry, as above demonſtrated, that
when the diameter in ſuch circumvolution
intercepts nothing of the circumference between
its ſelf and the right-line tangent, the dia-
meter is then coincident, and the ſame right-
line with the right-line tangent; and of the for-
mer angle therefore, hath left nothing at all,
becauſe of the coincidency of the two right-
lines, whoſe parts can in that poſiture have
no inclination one unto another: and there-
fore there is not ſo much as the leaſt angle of
Contact, or any other angle whatſoever, left
after this circumvolution: ſo as the whole
Argumentation is a long arrow out of a ſtrong
Bow, but quite beſides the mark.

It is a ſeeming weighty objection that which
is urged out of opticks, and the uſage in that
ſcience, to demonſtrate in conical figures,
the angles of incidence and reflexion to be
equal, only with reſpect to the right-line tan-
gent, touching the figure in the point of inci-
dence and reflexion, without ſpecial reſpect to
the curvature in the conical ſection. But hereto
without wrong, either to truth, nature, or
that noble ſcience, may be upon good grounds
anſwered. 1. That opticks is not pure Geome-
try, and obſtructed ſtereometry, and mathe-

maticks; wherein quantities, menfurations, and proportions are confidered merely as in themfelves without relation to matter, and the ufes to which in other faculties they are applicable: but in opticks, is an improvement of what in nature may be obferved about luminous and vifual beams, and luminous mediums and objects, by mathematical demonftration and affiftance. Now 2^{ly}. Nature doth not tye it felf in its wayes fo ftrictly to an indivifible, abfolute, vertical punctilio, that if it cannot reach, without impediments, that mathematical exquifitenes, it will not act at all. If heavy things on the clive fide of an hill cannot defcend in a direct line towards the Center, however they will feek it in the neareft oblique line poffibly they can, fo long as the motion brings them nearer: all animals are not formed in the fame mould, or with equal ftrength of Body, or vigour of fpirit. So as in thefe optical inftances, the brave *Authors* fell not in the leaft fhort of their task, when they had demonftrated the moft equal right-lined angles which nature could caft it felf into, in the alledged cafes, *Viz.* angles of incidence and reflexion equal, if compared with a right-line tangent at their point. For as elfewhere, they are but very few points in moft conical fections, where, by a right-line, their angularity can be divided into equal parts: nature therefore
when

when it cannot attain its prime and general
defign, which is a perfection and abfolutenefs
in its work, it doth not therefore flugge and
do nothing, but feeks to approach its firft
defign as near as poffible: and that is in ma-
king with right-lined beams equal angles re-
fpectively to the right-line tangents at the
points concerned; which angles fo formed
at thofe points with refpect unto the right-
line tangents, are as is elfewhere fhewn,
either the leaft of the greater right-lined an-
gles, or the greateft of the lefs right-lined an-
gles : that in fuch curvatures with great
judgement the quality of the angles of inci-
dence and reflexion, in beams paffing by right-
lines, as affected for their directnefs and fhort-
nefs, and as near as poffible endeavoured by
nature, is in demonftration referred to exami-
nation at the right-line tangents of the fame
points; an abfolute equality by right-lines to
be made, being moft what impoffible, and
that demonftrably in fuch curvatures : fo as
caufelefs was the exception which was made
againft the demonftrations of the noble Per-
fpectivifts : nor ftood, either they, or nature
in need of that improper, lame folution and
help, by making recto-convexe angles of
Contact to be neither angles, nor quanti-
tative.

The truth of the angularity and quantita-
tivenefs

venefs of recto-convexe angles of Contact thus
afferted; however, the reft of the things in
this controverfy cannot be determined without
a clear underftanding of what homogenealnefs
it is that is requifite for proportionals, and
which is mentioned in the definition of propor-
tion. To bring our felves unto the right under-
ftanding of which, know there are three wayes
whereby one thing may be faid to be greater
then another.

1ft. Improportionably, and by the whole
kind; as in heterogeneals: in which fometime
the leaft of the one is ftill, beyond all propor-
tion, greater then the greateft of the other: as
the leaft furface is greater then any line; for a
longer line may be drawn in the leaft furface,
then any the longeft line, that can poffibly be
given: and fo the leaft body is improportio-
nably, and heterogeneally, and by the whole
kind greater then any furface: for by the divi-
ding, or altering the figuration of the leaft
body, it may be made, by its perimetry, to
exceed the quantity of the greateft furface, that
poffibly can be given: fo a point, or a circle
of a foot diameter, is faid to be lefs then the
whole world; which is to be underftood, as
by the whole kind, and without proportiona-
blenefs. And fo I doubt not, but it will be con-
feffed, that the leaft recto-convexe angle of
Contact, is greater then the greateft acute,

right-

right or obtuſe right-lined angle : and the leaſt right-lined angle , then the greateſt recto-convexe angle of Contact.

2 y. One thing is greater then another infinitely , or , if you pleaſe , indefinitely : ſo an infinitely , or indefinitely inlarged line , is longer then any given line , without limitation , and ſo conſequenely without proportion. Or,

3 y. One thing is greater then another, rateably and according to the proportion , which they hold at the ſame ſtandard , whereby they are after the ſame manner meaſured , in the ſame kind in which they are compared. So one lenght , or line is longer then another , according to ſuch a proportion of length when both are meaſured as lengths at the ſame ſtandard upon an indefinite line ; and this , whether the lengths , or lines be , one , both , all , neither , or none of them . directly , or indirectly poſited , in right , or crooked lines ; and whether the one be a depth , and the other a height , or another a breadth , or a fourth a periphery. So one number is greater then another number, according to ſuch a proportion : and , though it is well known to be determined in Philoſophy , that numbers are of different kinds , yet for the proportion they hold one to another in their common nature , there cannot be denied unto them the truth of

it

a mathematical and analogical homogeneal-
nefs.

And as things are faid to be feveral wayes
one greater then another ; fo homogeneity
wants not its feveral acceptations. What homo-
geneity is , is not at all any where expreffely
defined in the mathematicks ; but we are left
at large , rationally to collect , what is by that
Term in thofe faculties to be underftood. The
moft ufual acceptation of the word homoge-
neity in Philofophy is to compare any divifi-
ble being with the parts into which it may be
divided : thofe things being faid to be homo-
geneal , which cannot be feparatingly divided
into parts of any other name , or nature then
the whole is : as the leaft feparable part of wa-
ter is faid to be water; of wine , wine : and the
leaft feparable part of a line , a line : and thofe
things are faid heterogeneal , which by poffibi-
lity may be feparatingly divided otherwife ;
Viz. into parts of different name and nature
from the whole. All which , as appears , hath
its dependance upon the fimilar , or diffimi-
lar natures of the whole and parts. So all fo-
lids , furfaces , lines , and plane angles may be
faid to be homogeneal ; for by dividing and
feparating them part from part , every part of
the folid is a folid, of the furface a furface, of the
line a line, and of the plane angle a plane angle:
and by taking number in a large fenfe , every
part

part of a number may be said to be a number.
But then if you descend lower in numbers and
angles , even and odde numbers , &c. and
right-lined and mixed-lined and crooked-li-
ned angles will scarce be able to defend their
homogeneity at this touch : for Even numbers
may be divided into parts that are odde : and
a right-lined plane angle may be severed into
parts whereof the one may be a recto-concave
angle of a semi-circle , the other a recto-con-
vexe angle of contact ; of which last mentioned
severed part you say , it is no angle , and there-
fore it must be heterogeneal : and we affert
it heterogeneal, because it is improportionable,
and its quantity not menfurable after the fame
manner , that the quantity of right-lined an-
gles is meafured.

And with reference to the fame acceptation
two diftinct beings , being compared together ,
as to homogeneity and heterogeneity , they
are then faid to be homogeneal , when both
having the fame name and nature , the one
may be the fevered part of the other , or both
together the fevered parts of an integral of the
fame both name and nature. So all folids are
homogeneal : all furfaces : all lines : all plane
angles and all numbers whatfoever.

But fometimes alfo in an abufive accepta-
tion , the agreeing , or difagreeing of things
in fome general , or fpecial kind and nature ,

paffeth for a certain kind of homogeneity or
heterogeneity among them : fo bodies and
furfaces and lines in refpect of their common
agreeing in quantity , continuity , longitude,
latitude and profundity , are partly homo-
geneal , partly heterogeneal : fo numbers in
refpect of their general quantitativenefs, parity,
imparity , fymmetry and a fymmetry &c. Are
partly homogeneal , partly heterogeneal ; and
fo folid , fphærical , and plane angles , and all
their feveral kinds are partly homogeneal , and
partly heterogeneal.

So as from any of the former acceptations
it is no way poffible to determine what is that
mathematical homogeneity which is every
where hinted unto us in the definitions of pro-
portion ; for the bolting forth of which we
fhall be conftrained of neceffity to betake our
felves to another courfe , in the queft of which,
it is not likely but we fhall meet with fome
abftrufenefs and difficulty,

To make therefore an Effay ; proportion
being the habitude of the compared magni-
tudes according to quantity , it is an homoge-
neity in quantity which is only herein requi-
red; not an homogeneity in fubftance , qua-
lity , fite , or other kinds and manners of being:
for heterogeneity in any of thofe other re-
fpects, as fubftance , quality , fite , or other
kinds of being , doth not hinder , but they may
not-

notwithſtanding be one proportionable to another, if any quantitative homogeneity be to be found amongſt them. So the heterogeneity of figure and figuration, which is between a Circle, pentagone, ſquare, triangle, ſurface of a pyramide, &c. Hinders not, but they may be compared and hold proportion in reſpect of their Area's and ſuperficial contents; wherein they retain a quantitative homogeneity : and thoſe of them which are meerly bounded planes, alſo have a homogeneity in reſpect of their perimetries, or the like quantities which they hold in common. So crookednes and rectitude are different kinds of poſiture, and make an heterogeneity in ſituation ; yet a right-line and a crooked-line, hold ſtill a quantitative homogeneity , in reſpect of their longitude and extenſion. So the different poſitures of the ſurfaces in a ſphære, and in a cone, and in a plane, hindereth not their analogy in Area and quantity.

But further to purſue this diquiſition ; though it be homogeneity in quantity , which is herein underſtood yet it is not the being quantities, or magnitudes in general that doth make up, or can compleat the homogeneity hinted in the definition of proportion : for then lines, and bodies, and numbers, and ſurfaces, being all of them magnitudes and quantities, there ſhould be a proportionablenes between lines

L   and

and bodies, bodies and surfaces, surfaces and numbers; which in such a general-homogeneity it were vanity to look for. In like manner, though plane, sphærical and solid angles be all angles, yet I think there are very few, that because of that their general homogeneity will expect a proportionablenes amongst them.

Besides as little necessity is there of restraining this homogeneity of magnitudes, unto their very last, and in every respect, ultimate kinds: except you will understand it to be in respect of some indefinite, most pure abstracted quantity, in which the compared magnitudes are to be each of them measured. For though some referr all numbers unto quantity, distinguishing them into several kinds, not only from their parities and imparities, with all their variations thereupon, but making every number to be of a different kind, receiving its specification from the last unite; as some also referr unto quantity all lines, which they distinguish into several kinds, & the like specifick distinctions are made by others of surfaces and bodies: yet between proportionals, for their homogeneity, is not, we confesse, in this sense required, that both magnitudes be of the very same ultimate kind: for hyperbolar, parabolar, elliptical, circular arches, and right-lines are homogeneal in length, extension, and as lines, so holding analogy, though in respect of rectitude and

cur-

curvature, and their feveral kinds of curvature
they be heterogeneal. So all numbers, even,
odde, commenfurable, or incommenfurable,
however, as above is faid, heterogeneal, yet as
multitudes and numbers in general, and the ac-
counts how oft an unite and its parts is pofited,
or how far in account is proceeded, whether re-
trogradely or progreffively from an unite; fo,
all numbers are homogeneal, holding mutual
analogy one to another. So notwithftanding the
diftinction between acute, right, and obtufe,
right-lined angles, all truth and exquifitenes of
proportion, as the meafure of their quantitative
relation, is moft apparent amongft them.

Which again doth further fhew unto us
what and how great is the difficulty of limi-
ting the homogeneity expreffed in the defini-
tion of proportion; fo as neither to fet it ufe-
lefly too high, nor fink it lower then is necef-
fary for proportionality. For right right-lined
angles are proportionate, one to another,
but there can be amongft them no proportion
of inequality: and no given acute right-lined
angle can have fo fmall a proportion to any
acute right-lined angle, as it may have and
hath to a right right-lined angle, or to any
obtufe right-lined angle: and though all even
numbers hold analogy and proportion one to
another, yet even numbers are not capable of
all diverfities of proportion; as in uneven

num-

numbers, duple and fubduple proportions are
imp ffible : nay numbers in general, I mean
unbroken and integral numbers, are not capa-
ble of all kinds of analogy ; as particularly not
of fuch proportions as are afymmetral.

But not to be nice in my thoughts to my
friend, I never in this point and queition under-
ftood any other thing to be meant by homo-
geneity in the definition of proportion, then
a menfurablenes of the quantities of two, or
more magnitudes in the fame indefinite quanti-
ty for kind, as the meafure of their quantity
and of the quantity of all their homogeneals,
the menfuration therein ftill being according
to that fame kind of quantity of which the in-
definite quantity is. And fo proportion is the
rate and habitude, which the rateable magni-
tudes hold mutually one to the other in refpect
of the fame way of meafuring their quantities,
or in refpect of the fame kind of indefinite
quantity in which their quantities are mea-
fured. And upon this glofs as the true and ge-
nuine meaning of this mathematical homoge-
neity, I ever underftood that poftulate to be
founded in which is required and granted, fo
to multiply any given quantity, as to exceed
any other given quantity whatfoever of the
fame kind. For if that menfurablenes in the
fame indefinite quantity, as a meafure, and
according to that fame kind of quantity of
which

which the indefinite meaſure is, were not the
very thing deſigned by mathematical homo-
geneity, the matter of the Poſtulat were not
fit to be granted without proof: for it is becauſe
they are meaſured in the ſame indefinite quan-
tity for kind, and according to the ſame kind
of quantity, *i. e.* they have the very ſame way
of meaſuring in the ſame indefinite quantity
(which is their homogeneity, by neceſſity of
conſequence creating a proportionablenes be-
tween them) that the leſs by multiplying
may be made greater then the greater, and
the greater by a continual cutting off ſtill more
then half may be made leſs then the leſs.

And though hereby homogeneity and pro-
portionablenes be not made to be the very
ſame thing, however in the mathematicks,
where the phyſical natures of things are not
inquired into, the one by a neceſſary conſe-
quence doth immediately flow from and is
annexed unto the other; and becauſe of their
neceſſary connexion, in uſual ſpeech and ac-
ceptation, the one may be allowed to be taken
for the other. And when in the definition of
proportion, proportion is ſaid to be the mu-
tual habitude of magnitudes of the ſame kind
according to quantity, or if you pleaſe multi-
plicity; the meaning is no other, but that pro-
portion is the mutual habitude of magnitudes,
which have their menſuration after the ſame

man-

manner, according to quantity, or multiplicity; taking the word multiplicity in a large
fenfe: *i.e.* according to the quantity and multiplicity, which they have each to other in
the fame indefinite quantity and meafure,
upon which they are in the fame manner and
according to the fame kind of quantity meafured. However in natural Philofophy for very
weighty reafons homogeneity and proportionablenes are to be acknowledged of very diftant and different natures. So I prefume in
Mathematicks it would be taken for a folæcifme to fay a body and a line were homogeneals and of the fame kind, becaufe all feparable parts of each agree in their being all
of them continuous quantities; though in the
Phyfiological fchool that they do concenter
and meet in the fame general nature is not deniable; and fo they may carry a feeming fhadow
of homogeneity, fo far as homogeneity may
be abufively wrefted to denote any fuch common agreement in a general notion and nature.
So it would be a folæcifme in Mathematicks
to fay, that a folid angle, a fphærical angle
and a plane angle were all homogeneal, becaufe they are all angles and every feparable
part of each is an angle: but to prove Mathematical homogeneity, the menfurablenes of
the quantity of the compared magnitudes in
the fame indefinite quantity, or meafure for
kind

kind , and according to the fame kind of quantity with the indefinite meafure is to be made out: for that all are quantities , or all angles, makes them not in the mathematical fchool homogeneal ; except by reafon of this menfurability of the quantity of both in the fame indefinite quantity , or meafure , according to the fame kind of quantity , the lefs by multiplying can be made greater then the greater , and the greater by dividing lefs then the lefs. And indeed this is the true homogeneity , not denoting a general conveniency in their natures in refpect of fome abftracted notion , but rather a fpecial identity by reafon of their menfurablenes in the fame fubftrate kind of quantity and meafure , only with difference , or proportionablenes of magnitude between both the wholes , and all the leaft , or greateft , proper , *i. e.* homogeneal parts of each : as if one be a line , fo is the other , and the greateft and leaft proper , *i. e.* homogeneal or homometral parts of each are lines and proportionable to either. For , if befides the menfurablenes of the quantity of the compared magnitudes in the fame indefinite quantity , or meafure , be not alfo added that condition , that in that indefinite quantity , or meafure they are alfo menfurable according to the fame quantity for kind , of which the indefinite meafure is , and fo confequently proportionable one to another ;

by

by nothing will it yet be determinable , whether
all angles be not homogeneal : for in every
angle , though of several kinds , every part of
each angle is an angle : nor will it be deter-
minable , whether all continuous quantities be
not homogeneal ; for every part even of hete-
rogeneal continuous qua. tities is a continuous
quantity : nor will it be determinable whether
all numbers be not homogeneal ; for that all
numbers are of the same kind will be found
a doctrine of very hard digestion any where
save in the mathematick school : yet accor-
ding to this explication of mathematical ho-
mogeneity , notwithstanding the diverlity of
their kinds in other philosophical considera-
tions , they have in them a clear mathema-
tical homogeneity : and even an unite , which
in other parts of Philosophy is not passable
for a number , will fall also within the verge
of the same homogeneity ; as will also all the
parts of an unite , whether commensurable ,
or incommensurable. And this explication of
mathematical homogeneity will be allowed its
due right and justification more easily , by
those who note how the main matter and de-
sign of mathematical definitions is but exege-
tical to clear up what is meant by the terms
in those sciences used : for what other occa-
sion could there be in the Mathematicks to
intermeddle with homogeneity , but to ex-
plain

plain the noble points of proportion and pro-
portionality.

And yet though in mathematicks there be
such a grand affinity between the proportio-
nablenes and homogeneity of magnitudes and
in common ufe and fpeech the one may be put
for the other , yet as above the notions are
eafily diftinguifhable by the underftanding: *Viz.*
two , or more magnitudes are faid homogeneal
chiefly in refpect of the fame way of meafuring
them , or in refpect of the fame kind of inde-
finite quantity in which they are both men-
furable : but they are faid proportionable in
refpect of the mutual habitude and quantita-
tive relation which is between themfelves
upon fuch their menfuration , in the fame way,
or according to the fame kind of quantity. So
all finite lines are homogeneal , as menfurable
in the fame indefinite line ; but that one finite
line is double to another is the habitude of
the one to the other declared upon that men-
furation.

That upon the whole matter mathematical
homogeneity doth not infift only upon iden-
tity in kind at large , or reftrain unto iden-
tity in kind at all points , and in every re-
fpect and confideration ; but it is identity in
quantity and therein particularly in the man-
ner of the menfuration of their quantities , in
which the homogeneity of magnitudes is chiefly
<div align="right">lodged :</div>

lodged: and that is that which I ever under-
stood in thofe words (according to quantity,
or if you pleafe, rather quotuplicity) which
are in the definition of proportion, *viz.* quan-
tity and quotuplicity in the fame way of mea-
furing.

And if you urge here, according to this
account I muft conclude all incommenfura-
bles to be heterogeneal; I anfwer, not in the
leaft: for though they may have no common
meafure, which can by poffibility exactly mea-
fure both or all, yet there may be a common
indefinite meafure, in which each may exactly
meafure forth its own quantity. As for example
in fig. 24. Let *A* and B be incommenfurable
lines, and DC. an indefinite line, beginning
at D, and on the part of C infinitely produ-
ced: from the point D in the line DC take
the line DE equal to the line A : and alfo
in the line DC take the line DF equal to the
line B. here in the line DC as a common in-
definite meafure of their homogeneity, the
two lines A and B, though incommenfurable,
have meafured forth themfelves by the lines
DE and DF. So in fig. 1.upon A the angular
point of the right-lined angle BAC, as Cen-
ter, draw the Arch dfe cutting the line AB
in the point d, and the line AC in the point f,
and the right-line Ae in the point e, fo
as the Arch df be incommenfurable to the
Arch

Arch fe. Here the two right-lined angles
dAf and fAe are incommenfurable , yet
have a common way of meafuring their quan-
tity and proportion , *Viz.* by Arches of Circles
drawn upon the angular point , as center ,
intercepted refpectively between the fides : as
by the *Arch* dfe , the one being in that
Arch meafured by the Arch , df , the other
by the Arch fe. So as by mathematical ho-
mogeneity is underftood an homometricalnes,
or autometricalnes with the neceffary confe-
quent of a rateablenes therein , without any
neceffity of fymmetricalnes between themfelves
at all. For ftill , if in homogeneity befides ho-
mometricalnes in the fame indefinite quantity
be not alfo included , that the menfuration
of both be according to the fame kind of quan-
tity , of which the indefinite quantity , in
which they are meafured , is ; a point and a
line may meafure themfelves in an indefinite
line , and a line and a furface may meafure
themfelves in an indefinite furface , and a
furface and a folid may meafure themfelves
in an indefinite folid. But true mathematical
homogeneity is when two , or more quanti-
ties being menfurable in the fame indefinite
quantity , or meafure, and according to the
kind of the indefinite quantity are by confe-
quence rateably, *i. e.* proportionably and ho-
mometrically equal , or the one bigger then
the

the other, not the one infinitely bigger than
the other, as an infinite line is longer then
a finite line, for between such it is acknow-
ledged there is no proportion nor mathema-
tical homogeneity ; nor the one bigger then
the other by the whole kind, as every folid is
bigger then any furface, and every right-lined
angle then any recto-convexe angle of Contact:
and proportion is the rate, or quantity of their
mutual habitude in that their homogeneity, or
homometricalnes : *i. e.* proportion is the rate,
quantity, or account of their proportionality.
And applying our minds rightly to conceive
of homogeneity as mathematical ; of neceffity
fuch, and no other can be the notion of it,
thereby making two quantities to be rateably,
i. e. proportionably equal, or the one bigger
then the other, and according to their capacity
and poffibility in any rate and proportion fo to
be conftituted and fet out. For the genuine well
known notion of homogeneity in general, what
is it but that all and each of the proper, *i. e.* ho-
mogeneal parts, and the whole, fall under the
fame Denomination and nature, as if one be a
line, all the reft to be lines, if one be water, or
ftone, or oyle, &c. all the reft of the parts and
the whole to be fo alfo ? And in the mathema-
ticks what is faid properly and homogeneally to
be a part of any magnitude, but only fuch leffer
magnitudes feparable from it as are able to mea-
<div align="right">fure</div>

fure out, *i.e.* by their multiple to exhauſt the firſt
magnitude ? For a ſurface is not ſaid to be an
homogeneally proper part of a ſolid : nor a line
of a ſurface. And by ſuch meaſuring forth of the
integral magnitude by its homogeneally proper,
i. e. exhauſting parts, is not underſtood the ſym-
metry of the parts, and whole, for the whole
and its true homogeneal parts, may be incom-
menſurable ; but that which is underſtood is
the true mathematical homogeneity of the
whole and the parts in reſpect of their com-
mon way of meaſuring, and the proportiona-
lity, which is thereupon lodged between them
So as by laying theſe undeniable remarks to-
gether, mathematical homogeneity in reſpect
of the ſame integral magnitude, is that every
part thereof being proper, *i. e.* menſurable in
the ſame indefinite meaſure according to the
ſpecial kind of its quantity, and therefore able
to exhauſt the whole, be all of them only in
reſpect of their quantitativenes, not their figu-
ration, or other reſpects, of the ſame nature
and denomination, and any proportion what-
ſoever (according to the capacity of ſuch kind
of quantities) poſſible to be conſtituted amongſt
them. And by conſequence ſeveral magni-
tudes are then ſaid to be mathematically homo-
geneal, when being the one able to exhauſt
the other, & ſo in proportion one unto ano-
ther, both and all the proper parts of each,
all

all of them , as being of the fame nature fall under the fame quantitative denomination.

And here it is worth our noting , how fome magnitudes are fo homogeneal , that they cannot be feparatingly divided into parts , or magnitudes which are heterogeneal ; but all the parts into which they are feparatingly divided will ftill be homogeneal , both one with another and with the whole : So a line can no way be feparatingly divided into parts which are magnitudes , but each is a line homogeneal to the whole and to all parts whatfoever of whatfoever other line : neither can a body and furface be feparatingly divided , but into bodies and furfaces ; each of whofe parts are ftill able to exhauft the whole and are thereunto proportionable. But fuch an abfolute homogeneity is not in all other magnitudes , efpecially in heterogeneally concrete magnitudes, but that the whole may be divideable into parts feparable one from another, which yet are heterogeneal : as in the prefent cafe of angles a right right-lined angle is divideable into a recto-convexe angle of Contact , and the recto-concave angle of the femi Circle ; which are feparable one from another, and therefore truly parts ; yet neither of them is homogeneal to the whole , nor one of them unto the other : for neither can the whole right right-lined angle , nor the recto-concave angle

of

of the femi Circle ever be exhaufted by any number whatfoever of fuch heterogeneal parts, as is the recto-convexe angle of Contact: nor ever any equality , or other proportion can poffibly be fhewn between the right right-lined angle , and the recto-concave angle of the femi Circle, becaufe there is no way poffible in which their quantities can be proportionably menfurable. For not without very good reafon unto all magnitudes are to be allowed their fpecial properties; as to all pofitures and figuration , theirs. To angles thefe things are peculiar, being otherwife in other magnitudes: *Viz.* in angles which are truly and on all hands confeffedly homogeneal , you cannot to any given angle, fet forth another of the fame kind in any given proportion at pleafure : for every right-lined angle by a neceffity of nature, muft be lefs then two right right-lined angles: and in a plane all the angularity at any point cannot exceed what the circumjacent fpace , or plane is capable of; which is only four right right-lined angles. That as number cannot be infinitely divided without fraction ; fo angularity cannot at pleafure at the fame point, in the fame plane , be inlarged : whereas fome other quantities have both infinite divifibility and infinite multiplicability.

So another property of the magnitude of angles
<div align="right">gles</div>

gles is, that it may not only in notion and fpe-
culation, but in truth and feveringly, be di-
vided into parts either able, or unable to ex-
hauft the whole: as when a right right-lined
angle is divided into the recto-concave angle of
a femi Circle, and a recto-convexe angle of
Contact: you may fever them the one from
the other: and angularity is equally, if not
much more apparent in the recto-convexe an-
gle of Contact, then in the recto-concave an-
gle of the femi Circle; yet the one of them is
demonftrated and confefled unable ever to ex-
hauft the right right-lined angle, the other
not.

A further property of the magnitude of an-
gles is, that fometimes the fame part, which
hath already been fevered from it, cannot ex-
actly and immediately again by its equal be
fevered from it on the fame fide, though the
remaining angle be by the whole kind greater.
So after a recto-convexe angle of Contact is
taken out of a right right-lined angle, there
cannot again immediately on the fame fide,
be fevered from the remaining angle another
angle equal to the recto-convexe angle of Con-
tact, which was before fevered from it. If it
can, let it be performed.

Alfo the divifibility which is in the magni-
tude of all Angles, though boundles and in-
finite, in fome however, leaves the dividing

as

of the Angle into two equal parts; impoſſible:
as notwithſtanding the perpetual diviſibility
of lines, the ſide and diameter of a ſquare are
left incommenſurable. So ſome other angles
may be divided into two equal parts; but it is
impoſſible to divide them into three equal parts:
as convexo-convexe angles of Contact, with
infinite other convexo-convexe angles and con-
cavo-concave angles being contained under
equal, uniform and anſwerable Arches.

To conſigne this point, the principal thing
we have laboured herein to dilucidate, & as
we doubt not have effected is, that mathema-
tical homogenealneſs is not an homogeneity of
all the parts whatſoever, that are in the magni-
tudes, which are homogeneal in reſpect of
ſome ſpecial way of meaſuring their quantities;
or an undivideablenes of ſuch homogeneal
magnitudes into parts otherwiſe heterogeneal
according to which acceptation the word is
chiefly taken in other parts of Philoſo-
phy : for there is no right-lined angle
whatſoever, nor any other angle whatſoe-
ver, but as is up and down herein ſhewn
may be ſeparatingly divided into heteroge-
neal parts : but mathematical homogeneity
is homogeneity in the way of meaſuring
the quantity of the compared magnitudes,
Sc. in the ſame indefinite meaſure and quan-
tity, and according to the kind of the
M   inde-

indefinite meafure , and which thereupon fol-
lows , a proportionality between them , in
refpect of their common way of meafuring : and
of this mathematical homogeneity , fair foot-
fteppings are to be found every where in the
deducing of thofe demonftrations which con-
cern proportions and proportionals. That,
fuch magnitudes as have no common way of
meafuring their quantity , as weights and mea-
fures , are heterogeneal : or if they have a com-
mon way of meafuring in which they may
meafure themfelves , but therein do not mea-
fure themfelves according to the fame kind of
quantity with the indefinite meafure , and fo
want proportionality , yet notwithftanding
they are heterogeneal : as all recto-convexe an-
gles of Contact , all recto-concave angles of
femi Circles , all recto-convexe angles of femi
Circles , all acute , or right right-lined angles,
thefe may all meafure themfelves , and in
what order their fides fall within , or without
in any obtufe right-lined angle whatfoever :
yet becaufe this their homometry is only of
the fituation , or order , in which the fides part
from the angular point , but not of their quan-
tity in an indefinite meafure and according to
the denomination of the fame quantitative mea-
fure , fo as to lodge a proportionality between
the magnitudes fo compared together in their
common way of meafuring; they are not , nor
can

can thereby be vindicated from their otherwife innate mathematical heterogeneity : which concerning fome of them is confeffed on all hands and is without the verge of the controverfy. And as follows, angles are of a concrete nature, having in them fomething quantitative and fomething not quantitative : whereas that which is to be the indefinite meafure of homogeneal quantities is to be confidered abftraly as quantity without heterogeneal concretion : fo it is the circumference of a Circle that meafures all right-lined angles. And when all plane angles are faid to be homogeneal, it is not in refpe of a common indefinite quantity by which they are all meafured, which the reo-convexe angles of Conta doe fufficiently evince, but as is manifeft it is only becaufe of the pofition and fituation of the fides in the fame plane ; which homogeneity is of no concern unto quantity, nor by any neceffity can thereupon infer the confequent of proportionablenefs.

But to proceed ; as is faid, befides the former mathematical and quantitative homogeneity and heterogeneity there is alfo an extramathematical and extra-quantitative homogenealnes and heterogenealnes in angles every where obfervable in their fhapes, figures, pofiture of their fides, fuch like fchematifmes and other refpes. In general as is above hinted, every part of a

plane

plane angle is a plane angle, even the recto-
convexe angle of Contact, however you deny
it to be an angle and quantitative: but then
this is not a mathematical homogeneity, but
only in respect of a certain figuration, in re-
spect of the positure and situation of the sur-
face in which those angles are; shewing how
all plane angles from the greatest to the least,
agree in that particular of their general figura-
tion, *Viz.* of having their containing sides to
lye still in the same plane ; whereby they di-
stinguish themselves from all other superficial
angles, which are heterepipedal, whose con-
taining lines, or sides lye in several planes:
such as are all sorts of sphærical, cylindrical
and conical surface-angles. But if ever a mathe-
matical and quantitative homogeneity be pro-
ved among all plane angles, you that know
that it is not my use to start from my word,
shall hereby rest assured, upon the first sum-
mons I will give up this cause.

And we are not to think strange , that a
figuration is asserted to be in angles ; for if
we seriously consider , weeshall find there is
shape and figure in angles, as well as quantity;
as lines, and surfaces, and bodies have their fi-
gurations, the positure of their parts, their
shapes and forms , as well as their quantities
and magnitudes : in each , their figuration
being manifest; *Viz.* in lines, in respect of
their

their lineary pofiture : in furfaces, in refpect
of their fuperficial pofiture : in bodies in re-
fpect of their folid pofiture : and in the cafting
of each of their fchematifmes quantity is invol-
ved, as length, breadth, depth, *Viz.* their
quantities, and the quantities which are com-
pounded of feveral, or all of them together.

And here by the figuration which we affert
in angles, we cannot be thought to mean
that any right-lined figure can be compleated,
perfectly to bound up a plane on all fides,
by one angle ; it being beyond the power of
two : and three, being the leaft number of
angles, requifite fo to conftitute and perfect-
ly limite out a right-lined plane figure. *And*
though fome plane figures are perfected,
and perfectly bounded, without any fuch
angles as are contained by fides concur-
ring by way either of fection, or Contact,
as namly all Circles and Ellipfes ; yet the
angularity of curve coincidence is every where
found, or at pleafure affignable in the boun-
ders of fuch figures. But our meaning is, a plane
angle, though moft what it do not by the
continuation and production of its fides, per-
fectly bound in and limit out a certain plane
and fpace on e vry fide, however being the
mutual habitude of concurring lines, it gives
an imperfect figuration to the plane and fpace
on its part. *And* as a bounded plane cannot be

M 3   with-

without fome kind of plane figure, fo a limited angle ever implies in it an imperfect figuration of fome fort, or other. For figuration is the confectary of material finitenefs and limitation in the pofition of lengths, breadths, depths, furfaces and folidities; that every angle having its limits and bounds cannot be thereof deftitute. And if the name of figures be fo frequently given to hyperbola's, parabola's, and the like, which neither do, nor ever can by any poffible production perfectly bound in their planes; what reafon is there then why angles fhould be denied an imperfect intereft in the name. Befides as a plane in its own general nature at large doth not denote any fpecial plane figure; but the rife of figures, I mean plane figures, is from the bounding of the plane: fo it is in angles, as they by the mutual habitude of their concurring fides give imperfect limits and bounds unto the fpace and plane, fo they therein make an imperfect figuration. That in angles fomething of form and figure is to be noted as well as magnitude. And one line cannot concurre with and be inclined upon another, but an imperfect figuration will arife from that their mutual inclination. And the fame two angles may have the inclination, *i. e.* the receffe of their refpective fides one from another equal, though there be no analogy between the figurations

of

of the angles, or the fhapes in the which the
fides are inclined in the one and in the other.
For by reafon that in angles form and figure
are to be obferved as well as quantity ; crook-
ed-lined and right-lined angles may be equal
in fome particular quantity, yet other-wife not
of the fame kind : they having equality in fome
magnitude , but being diftinct in the man-
ner of their forming, figuration and conftitu-
tion : as equality may be between a fquare
and a triangle, though figures altogether dif-
ferent in kind.

And in refpect of fuch their figurations,
plane angles receive diftinction , either from
the diverfe manners in which their contai-
ning fides do concurre , or elfe from the
diverfe natures , and figurations of the lines
under which they are contained , or ,
which is tantamount from the diverfity of the
inclinations and inflexions, or rather inclina-
bleneffes and imflexibleneffes by which they
are inclined each to other , or from feveral
of thefe grounds of diftinction taken together.

Plane angles from the different manner of
their fides concurring , may aptly be thus dif-
tinguifhed , *Viz.* into angles, whofe fides con-
curre by way of fection ; or elfe that have their
fides concurring only by way of touch in fome
fingle fingular point without mutual fection ;
or elfe their concurrence is in curvature , where,

after

after the meeting of their sides in the angular point, the sides do not in their productions depart one from another, neither by way of touch, nor section; but become, the production of the one side coincident with the other side; so as this kind of angles may aptly be called angles of coincidency, or angles of curvature: and in these lies the genuine Ratio and true account of the curvature.

From the diverse figuration of the lines, under which a plane angle is contained, very many differences of angles may arise, according to the various distinctions, of which lines themselves are capable, I mean such lines, as fall not without the capacity and comprehensivenes of the same plane: as that some are helicoidal, some parabolar, some elliptical, &c. But as of lines, so hence of angles, the chief and primary distinctions are especially these, *viz.* that plane angles are either right-lined angles, contained under two inclined right-lines; or not right-lined angles.

Not right-lined angles, are either mixed lined angles; contained under one right-line, and one crooked-line: or crooked-lined angles contained under two crooked-lined sides. And from the several kinds of special, or ordinary curvatures, as Circular, elliptical, hyperbolar, &c. The mixt-lined, and crooked-lined angles, are capable of many farther and

more

more particular diftinctions ; but efpecially
from the fite of the convexenefs, or concave-
nefs of the lines to , or from the angle fide:
though all fuch fecondary diftinctions rifing
from thefe two laft mentioned heads , are as
properly and pertinently referable to the other
ground of diftinguifhing plane angles, taken
from the differences which may be in the in-
clinations of the one containing fide to the
other. For a vaft difference is in the incli-
nation of a crooked-line , by obverting the con-
cave , or convexe fide to any other line. So
the conftituting a circular , or elliptical arch
&c. For one fide , makes a vaft difference in
the inclinations , becaufe of the difference in
their curvatures.

Alfo another principal diftinction of angles
from their fides, may be into angles, whofe
fides are coaptable , and by poffibility may be
coincident one with another : or elfe fuch as
have between them no poffibility of coapta-
tion and coincidence. Of the former fort are
all right-lined angles , and all concavo-convexe
angles contained by Arches of equal homoge-
neal, uniform, regular , or anfwering curvatures:
of the later fort are all other ; whether mixt
hed angles , or crooked-lined angles ; whether
they be mixed crooked-lined angles , or unmixt
crooked-lined angles. And confequently there-
upon befides the numerous diftinctions of an-
gles

gles in refpect of their different inclinations,
fuch as above mentioned ; one is more emi-
nently material above the reft , that the incli-
nation of the fides , is fometimes with an equa-
bility all along their production ; though ima-
gined to be infinitely extended , in fuch lines
as by poffibility may with reafon be imagined
fo to be : and fometimes there is nothing of
equability to be found in the inclination of
the feveral parts of each fide to the other ;
though it may be one of the fides be a right-
line , or an Arch of moft equal , uniform , re-
gular and homogeneal curvature. And this
equability and inequability of the inclination
of the fides , ftrangely alters the properties of
angles. As in right-lined angles , for the equa-
bility of the inclination of the fides , no parts
of the one fide are more inclined then the
reft unto the other fide : and fo in concavo-con-
vexe angles of equal curvatures , no parts of
the one Arch are more inclined then the reft
unto the other ; but the one Arch is all along
inclined unto the other , as at the angular
point ; and the inclination , which the one
bears unto the other at the angular point , is
obvioufly expreffable as to the quantity of the
receffes , which they make one from anothe
by the inclination of a right-line to a rigʰᵗ
line , except when the inclination of the Arᶜˢ
is equal to, or greater then two right rightᵉᵈ
                                                      an-

angles. And in such crooked-lined angles, whose
sides have equability of inclination, the points,
which from the angular point are at equal di-
stances along the Arches, are also absolutely
at equal distance from the angular point, along
the chords: and right-lined tangents at any two
such homologal points, where ever taken, al-
wayes meet and contain a right-lined angle,
equal as to the quantities of the recesses of the
sides, to the crooked-lined angle contained by
the two Arches; as is obvious to demonstrate,
especially in circular Arches. And the right-
lined angle contained under the two right-lined
tangents touching at the two homologal and
answering points, which is equal to the iso-
clitical crooked-lined angle, if the two right-
lined tangents occurre on that side of the right-
line connecting the homologal points on which
the isoclitical angle falls, then it is the angle
contained by the two right-lined tangents into
whose space part of the space comprized be-
tween the two Arches at first falls, which is
equal to the crooked-lined isoclitical angle :
but if they occurre on the other side of the
right-line connecting the two homologal points,
i. e. averfely from the crooked-line isoclitical
angle, then it is the complement of such an
angle, which is equal to the crooked-line iso-
clitical angle : but if the two right-lined tan-
gents occurre in one of the homologal points,
the

the angles either way contained under the
two right line tangents are equal, *viz.* right
right-lined angles ; either of them making
forth what is herein afferted. As in fig: 19. un-
der the two circular ifoclitical arches bda and
acn let there be conftituted the ifoclitical
angle b a c ; and let the right-line a g touch
the Arch a c n in the point a : and let the
right-line af touch the Arch adb in the
point a : fo making the right-lined angle f a g
equal to the ifoclitical concavo-convexe angle
b a c. Then take in the *A*rch a d b any point
at pleafure , *Viz.* the point d: and draw the
chord a d. Then in the Arch a c n take the
*A*rch a c fubtended by the chord a c equal to
the chord a d. Therefore becaufe of the ifo-
cliticalnefs of the circular Arches the two points
d and c are two homologal, *i. e.* anfwering
points the one in the one *A*rch, the other in
the other, *Viz.* the point d in the Arch a d b
and the point c in the Arch a c n. Then draw
the right-line d c connecting the two homo-
logal points d and c. *A*lfo draw the right-line
d e touching the arch a d b in the point d; and
the right-line c e touching the *A*rch a c n in
the point c. *A*nd let d e and c e the two right-
line tangents be produced till they meet in
the point e; which in this figure is on that
fide of the right-line d c on which the con-
cavo-convexe angle c a b lyeth. I fay there-
fore

fore that the right-lined angle d e c contained
under the two right-lined tangents d e and c e
touching the Arches respectively at the homo-
logal points d and c is equal to the right-lined
angle f a g, contained under the two right-line
tangents f a and g a touching the Arches re-
spectively at a the angular point of the isocli-
tical concavo-convexe angle. For the right-
lined tangent f a cutting d e the other right-
lined tangent of the same Arch a d b in the
point h; and the right-lined tangent d e of
the Arch a d b cutting the chord a c in the
point k; upon this constructi̇on the right-line
d a is equal to the right-line a c; and the
right-line tangent d h is equal to the right-
line tangent h a: therefore the right-lined
angle a d h is equal to the right-lined angle
d a h and so to the right-lined angles a c e
and c a g severally. And therefore the right-
lined angle a h e being equal to the two right-
lined angles h d a and d a h taken together,
and the right-lined angle h d a being equal to
the right-lined angle c a g; the right-lined an-
gle a h e is equal to the two right-lined an-
gles c a g and d a h taken together. Therefore
that which maketh each equal to two right
right-lined angles ; the two right-lined angles
h k a and h a k taken together are equal to
the two right-lined angles h a k and d a l ta-
ken together. Therefore the right-lined angle

h k a

h k a is equal to the right-lined angle d a l.
Therefore the right-lined angle c k e is equal
to the right-lined angle d a l. And therefore
that which makes either equal to two right
right-lined angles, the two right-lined angles
k c e and k e c together taken are equal to
the right-lined angle d a g , which is equal
to the two right-lined angles d a c and c a g
taken together : and the right-lined angle c a g
is 'equal to the right-lined angle k c e : there-
fore the right-lined angle k e c is equal to
the right-lined angle d a c: therefore becaufe
the right-lined angles d a h and c a g are e-
qual ; alfo the right-lined angle k e c Sc. d e c
is equal to the right-lined angle h a g Sc. f a g,
which was to be demonftrated.

But if the two right-lined tangents d e and
c e as in fig. 20. do not occurre towards the
concavo-convexe angle b a c, but on the other
fide of the right-line d c in the point e; then
is the right lined angle d e c contained under
the two right line tangents d e and c e touch-
ing at the homologal points d and c, not equal
to the concavo-convexe ifoclitical angle b a c,
or the right lined angle , equal unto it , f a g,
but to its complement unto two right right-
lined angles, *Viz.* unto the right lined angle
f a l; the right line l a being the produćtion of
the right line g a. For as before by conftru-
ćtion , the right line chords d a and c a to the

homologal points d and c are equal:and the right line f a cutting the right line d e produced in the point h , the right lines d h and a h being two right lines tangents of the same Circle a d b , occurring, are equal. And let the right line a c produced, occurre with the right line d e produced, in the point k. As appears;the right lined angles a d h , d a h and c a g , as before , are equal ; and the right lined angle a h e is equal to two right right lined angles , all but the two right lined angles h d a and h a d that is, all but the two right lined angles c a g and h a d. Therefore the right lined angle a h e being equal to the two right lined angles h k a and k a h ; the two right lined angles h k a and k a h are equal to two right right lined angles all but the two right lined angles c a g and h a d. Therefore two right right-lined angles are equal to the four right lined angles c a g and h a d and h k a and k a h. Therefore out of equals taking equals , the right lined angle h k a, which is the right lined angle c k e is equal to the right lined angle d a l. Therefore what on either side remains to make up two right right lined angles on either part ; the two right lined angles k c e and k e e are together equal to the right lined angle d a g, which is equal to the two right lined angles d a c and c a g taken together. And producing the right line c c till it cut the right line a g in the

<div align="right">point</div>

point $\overline{g}$; the right lined angles c a g and a c $\overline{g}$ and k c e are equal. Therefore the right lined angle k e c is equal to the right lined angle d a c. And becaufe the right-lined angle d a h and c a g are equal , therefore adding the common angle f a c, the two right lined angles d a c and h a g are equal. And therefore the right lined angle k e c is equal to the right lined angle h a g. And therefore their complements unto two right right lined angles , *Viz.* the two right lined angles d c c and h a l are equal ; which was to be demonftrated.

So in fig. 22. if the right line tangent d e paffe directly unto the other homologal point c, as it doth when the ifoclitical concavo-convexe angle is equal to a right right lined angle, and the homologal points d and c are taken at quadrantal or other diftances from the angular point ; then moft manifeftly the rightlined angles d c g and f a g are equal , being under the two and two refpective right lined tangents ; and between the angles and their complements unto two right right lined angles is no difference, as appears by what is in the former demonftrations. So in fig. 23. if the homologal points d and c be fo taken , that the chords d a and c a be the diameters : then producing the right line tangent c e till it occurre with the other right line tangent d e in the point e, and with the other right-line tangent

gent f a in the point f; the right lined angles
c f a and f a g are equal. And in the trape-
zium d e f a the two right lined angles e d a
and d a f are each of them a right right-lined
angle, therefore the right-lined angle d e c is
the complement of the right lined angle e f a
unto two right right-lined angles, *i. e.* it is
the complement of the angle f a g unto two
right right lined angles; which was to be de-
monstrated. And like demonstrations may be
formed upon every other case. But where the
isoclitical arches are not circular the demon-
strations must vary according to the prop'iety
of every several curvature, notwithstanding
its equability and isocliticalnefs.

But to return, in angles contained by sides
whose inclination each to other is without
any such equability, (except only in unmixt
crooked-lined anisoclitical angles) the account
of the homologal points along the chords and
along the arches is still different: but in none
of them do the right line tangents from the
two and two homologal points, still in their
meeting make the two same angles which are
made by the two right lined tangents at the
angular point; nor can any right line angle ex-
prefle the inclination, which the sides have
each to other at the angular point.

Hence is manifest how the same angle may
from several of the grounds of distinction here

N   pro-

proposed be referable to several heads, or kinds.
So right lined angles as isoclitical, have al-
wayes equability of inclination; and the con-
currence of their sides is always by way of
section and cannot be by Contact, or coin-
cidency. So mixed lined angles being anisocli-
tical have always inequability in the inclina-
tion of their sides; and their concurrence may
be either by Contact, or Section, but never
by coincidence. And in crooked lined angles,
their sides may have either equability, or ine-
quability of inclination : and accordingly the
concurrence of the sides may be by section, or
Contact; and with, or without possibility of
coincidence. Sc. Isoclitical crooked-lines, but
they must be posited convexo-convexely, or
anisoclitical crooked-lines posited whether con-
vexo-convexely, or concavo-convexely, or a
right-line and a crooked line, any two of
these may touch without cutting, and so the
angular sides have inclination tangent and not
secant, So isoclitical concavo-convexe angles
may have sides circumducted to coincidence:
but the same sides posited concavo-concavely,
or convexo-convexely become anisoclitical in
the circumduction, one in respect of the other;
yet either is isoclitical sometimes and in some
cases with the production of the other.

From these things though distinguishing an-
gles into their several kinds, only with respect
<div align="right">to</div>

to the diverfity of their figuration, may how-
ever more abundantly appear how unma-
nageable a task they take upon themfelves,
who to exclude recto-convexe angles of Con-
tact from being angles, and from quantitave-
nefs, would force all plane angles to be of the
fame kind, allowing no fpecifick difference
poffible among them. Not here to purfue
what other diverfities in kind may be obferved
among angles; how can the inclination of a
crooked-line upon a right-line differ lefs then
in kind from the inclination of a right-line
upon a right-line? For as a right-line and a
crooked-line agree as lenghts and in lineari-
nefs and are therein mathematically homoge-
neal, but as right and crooked differ in kind
and have therein heterogeneal figuration; the
crookednefs of the crooked-line having no
analogy to the rectitude of the right-line, nor
the rectitude of the right line any proportio-
tablenefs unto the curvature of the crooked-
line; fo the inclination of a crooked line upon
a right line, and of a right line upon a right
line they agree in the common nature of incli-
nation, and the one may be greater and the
other lefs as the refpective fides fall within, or
without: but this relation of greater, or lefs
is without any proportionablenefs, and only
by the whole kind, depending upon the paffing
of the refpective fides, the one within, or

N 2   without,

without the other. For thefe two inclinations
differ fo far in kind that neither the curvatu-
re of the mixt lined inclination hath any thing
in it conform, or proportionable to the re-
ctitude of the right-lined inclination, nor
the rectitude of the right-lined. inclina-
tion to the curvature of the mixt lined inclina-
tion. In a word, fo different is the inclinable-
ness of a crooked line upon a right line, from
the inclinableness of a right line upon a right
line, that it is impoffible for the one ever to be
either equal, or any way determinately pro-
portionable unto the other: becaufe the coap-
tation of a right line as a right line, to a crook-
ed line, as a crooked line is againft the proper-
ties of their figurations, kinds and natures. And
for what reafon fhould there be leffer difference
between a crooked-lined inclination and a right
lined inclination, then there is between a crook-
ed-line and a right line? Yet all this their di-
ftinctnefs concerns only an heterogeneity in
their figuration and not at all, or not primarily
their quantities.

The argument, if they be angles, or plane
angles, they are homogeneal and of the fame
kind, is of no more force then this confequence;
if they be quantities, or continuous magni-
tudes, they are homogeneal and of the fame
kind. And they that deny all heterogeneity in
angles, becaufe they are all angles; will find
it

it an hard task upon the fame ground to main-
tain an analogous homogeneity, or any other
confiderable homogeneity, between right lined
angles and fphærical angles, or any other an-
gles, made by planes cutting the heterepipe-
dal furfaces of folids, and efpecially folid an-
gles, of what fort foever. And to yield that
all plane angles are homogeneal; for it is true:
and the moft abfolute, proper and genuine ho-
mogeneity is among plane angles.: *i. e.* no part
of a plane angle can be any other then a plane
angle, how great, or little foever, and whe-
ther proportionable, or improportionable one
to another, *i. e.* whether mathematically ho-
mogeneal, or heterogeneal: yet if we ferioufly
confider what is this homogeneity which is
among plane angles, that all their parts are
plane angles, it is not as is faid any quantita-
tive, or mathematical homogeneity, the contra-
ry of which is plentifully demonftrated in Geo-
metry to be poffible; nor any fuch homoge-
neity in refpect of the manner of their pofi-
ture and figuration, as to exclude all farther
diftinguifhablenefs in refpect of fignration; but
only denotes that in every plane angle, and
in every part of every plane angle, the fides lye
ftill in the fame plane: which homogeneity,
as is plain, excludes neither heterogeneity,
in refpect of figuration, nor in refpect of pro-
portion and identity in the way of meafuring
their quantities.   N 3   To

To the objection that in fig. 12. the recto-
convexe angle of Contact BAF can be added
to the right right-lined angle BAG, so ma-
king the outer angle of a semi Circle F*AG*;
or taken out of it, &c. the recto-convexe angle of
Contact BAD out of the same right right-
lined angle BAG, so making the inner angle
of a semi Circle GAD : and that therefore the
recto-convexe angles of Contact F*A*B and B*A*D
and the right right-lined angle BAG, and the
two angles of the semi Circle GAD, and FAG,
are all of them homogeneal and of the same
kind; I answer. First what need is there of
such endeavours, for you to prove their ho-
mogeneity, it being Geometrically demonstra-
ted and confessed that there is no analogy, or
proportion between them ; I mean between
the recto-convexe angles of Contact and either
the right right lined angle , or either of the
angles of the semi Circles? And according to
your opinion that which is added or taken out
is said to be nothing. But especially its thought
strange, why there should be such doubting ,
that heterogeneals can be added and laid up
togetheras into one repository; it being with
as easy connexion performable , as is usual in
the addition of incommensurables and specious
quantities of which it is not known , whether
they be homogeneal, or heterogeneal. And
out of an heterogeneal sum , as a store-house,
why

why cannot fome of the heterogeneals be fub-
ducted, the reft remaining? and what is more
ufual then the adding of heterogeneal figures
one unto another? and fubducting out of a
given figure fome other figure, which is quite
heterogeneal to the firft given figure? So to
adde together numbers, and meafures, and
weights? the fum of which may be divided,
multiplyed, increafed, or leffened, notwith-
ftanding its heterogeneity. As fuppofing A
B, C, D, all heterogeneal, as is ufual in ana-
lyticks, the half, or third part, or any pro-
portionable part of this heterogeneal fum
may be given: and any one of the heteroge-
neal magnitudes fubducted, the reft remaining:
or a fifth heterogeneal magnitude added to the
former fum: or any Algorythme, ever fpe-
cioufly, fometimes compleatly and abfolute-
ly thereupon performed.

Befides upon geometrical demonftration and
your own confeffion, all recto-convexe, con-
vexo-convexe and citradiametral concavo-con-
vexe angles of Contact muft neceffarily by your
own principles be allowed to be abfolutely
heterogeneal to all right-lined angles what-
foever; your felf acknowledging that neither
in equality, nor in any kind of multiplicity,
or fubmultiplicity is any proportionablenefs
poffible amongft them. And where between an-
gles a mathematical homogeneity is confeffed
and

and allowed, yet heterogeneity in respect of
their Schematismes and figurations is undenia-
ble.

The things therefore constituting and distin-
guishing plane angles in respect of their figu-
ration are, as above, their sides, their incli-
nations, or rather inclineablenesses, and their
concurrence. That when two angles have all
these in the same respective kinds, the angles
are upon good reason in this sense concluded
to be homogeneal : but when between two an-
gles is an heterogeneity in any of these things,
which are of the essence and constitution of an
angle, those angles may justly be judged in
this sense to be heterogeneal. And that such a
specifick heterogeneity may be in each of these,
may easily be declared as above. As first in lines
which are the containing sides, how easy is it
to discover such an heterogeneity ? For though
a right line, and a crooked-line agree unde-
niably in the general nature of a line, and of
length, and of extension, yet the rectitude of
the one and the curvature of the other, are
several kinds of positure, into which the length
of the one, and of the other is disposed : that
except, in contrarieties, we can see nothing
but homogeneity, such an heterogeneity must
needs be acknowledged between them. And
whereas homogeneity, as to sides, inclination
and concurrence, is required to the homoge-
neal

neal figuration of angles, the heterogeneity of
the fides hinders the poffibility of ever making
them out to be fuch; or that by any altering
their divarication, keeping their prefent pro-
perties they can be coaptable. And that angles
contained by heterogeneal fides may be equal,
proves only the equality of the inclinations in
either, but not the homogeneity of the figu-
ration of the angles, or inclinations; as the
equality between a fquare and triangle in re-
fpect of their equal perimeters, area's, heights,
bafes, proves not in the leaft the homogeneity
of their figures. And as right-lines and crook-
ed-lines are heterogeneal, as above, not poffi-
bly to be coapted, with the precedent limita-
tions; So alfo are all curve lines, whofe curva-
ture is unequal and unlike, nay though it may
be they be but feveral parts of the fame line, or
though the curvature of both, be every way, and
every where equal and like, yet if the convexe
and concave parts of the one be not alike pofited
as in the other, there will be a manifeft hete-
rogeneity in them, and an impoffibility of coapt-
ing them, obferving the limitations as above.

And why doth the heterogeneity in the fides
make heterogeneity in the angles, but becaufe
thereby is founded an heterogeneity in the in-
clinations, or rather in the inclinableneffes of
the one fide to the other? For here, it is not
the feveral degrees of the fame kind of inclina-
tions

tion that is intended ; for then all unequal right-lined angles fhould be altogether heterogeneal one to another : but it is a more then gradual , a fpecifick diftinctnefs in their inclinableneffes , which we are now difcovering to make the figuration of the angles more fairly and fully heterogeneal. And as inclination is the habitude of line to line , not being pofited in the fame right line , nor parallel , for even perpendiculars are in this fenfe here faid to be inclined ; fo , as above , from the heterogeneity of the lines will arife an heterogeneity of inclinations : and indeed for no other reafons do heterogeneal lines make heterogeneal angles , but becaufe their inclinations are neceffarily heterogeneal. And , as above , heterogeneal inclinations being refpectively equal , as in fome right lined and crooked lined angles , this doth not in the leaft annul the heterogeneity of their inclinations ; as a right line and a crooked line may be equal , yet as to the pofiture of their extenfion they are heterogeneal.

. And as heterogeneity of fides , or inclinations makes heterogeneity of angles , fo likewife doth any heterogenealnes in the other point requifite to the nature of an angle ; which is the manner of the fides concurrence. And there are only three wayes in the concurrence of the fides of angles , according to which they can be heterogeneal one to another. For either the

pro-

production of the one concurring fide becomes coincident with the other concurring fide , or elfe it departs from it on the fame fide on which it did occurre ; or elfe it departs from it on the contrary fide to that on which it did occurre : all which are clearly not feveral degrees of the fame manner of concurrence , but feveral kinds of concurrence : *Viz.* the one by way of Contact , the other by way of Section , and a third by way of curvature , or coincidence. That as thefe are diverfified in angles , I mean from kind to kind , not from degree to degree , fo there is thereby lodged in their figurations an heterogeneity , though in fome mathematical refpects , neither fides , nor inclinations can fometimes be denyed to be however homogeneal. So particularly angles of Contact in refpect of their figuration muft neceffarily be acknowledged clear of another kind, then all other angles : becaufe the inclination of their fides is tangent , concurring only in a punctual touch , whereas the inclination of the fides of all other angles is fecant, and at the point of their concurrence by reafon of their inclination they cut one another, or elfe they are coincident; then which, what can make a more material difference in the inclination of the fides ? And as more efpecially relating to that fo much urged analogy between right-lined angles and angles of Contact; the inclination

of

of the conteining sides in every angle of Con-
tact is such as is impossible to be between the
sides of any right-lined angle : for the sides of
no right-lined angle can touch without cut-
ting. And what more manifest and material
difference can be in the inclinations made upon,
or unto a right-line, then if in the one case
a right-line be inclined unto it, and in the
other a circumference, or other crooked-line?
Yet further to clear that difference of kind,
which is between angles of Contact and other
angles ; I think on all sides it will be judged
unreasonable to make those angles of the same
kind; which have neither one common way
of measuring, nor are coaptable, nor any way
proportionable one to another , nor can any
way by the contraction, or dilatation of their
sides be made equal one to another; and this
we shall find to be the condition of many angles
one in relation to another. However mis-under-
stand me not, as if I made any commensurabi-
lity a full mark of a full homogeneity : for as
before crooked-lined and right-lined angles
may be equal and of different kinds; having
their inclinations different in the kinds of their
figurations, though equal in the recesses of the
sides.

And thus having at large deduced the grand
difference which is between the mathematical
heterogeneity of angles, and their heteroge-
neity

neity in respect of their figurations, it will now be easy for us to extricate our selves out of all the difficulties with which former Disquisitions upon this subject have been involved.

As first what is to be understood by the equality, which is asserted to be between right-lined, and isoclitical concavo-convexe angles. For it is out of controversy and on all hands yielded, that to any right-lined angle given may be given also a concavo-convexe isoclitical angle equal ; and that also, in a thousand varieties ; as is most manifest, in the circumferences of any two and two equal Circles, or any two and two equal Arches. And so in a converse manner ; to any isoclitical concavo-convexe angle given, whose sides make their recesse one from the other by an Arch less then a semi Circle, may be given an equal right-lined angle: although in the infinite number of right lined angles, it is impossible to find any more then one right-lined angle equal to the given concavo-convexe angle ; because in rectitude there can be no diversity, as there may and is in curvatures. Now in the above recited cases, why is equality between such different angles asserted possible ? and what is meant by their equality ? and whence, and how is the equality of them to be demonstrated ? Of necessity it must be founded upon some special method of measuring angles, or of somewhat which

which is in some, if not in all angles; of which in common both these different sorts of angles are naturally and indifferently capable. And to be short, particular and plain; all the mysteriousness of this their equality is founded upon this: that these two sorts of angles, right lined angles, and concavo-convexe angles of equal arches, they both have in common one special property, of which all other sorts of plane angles whatsoever are destitute. *Viz.* that each in their kind are isoclitical angles, and the sides in each are isoclitical, and in each angle the one side by the adduction, contraction, and drawing together of the sides will be coincident and coapted unto the other. And as the coincidence of right lines the one upon the other makes a right lined angle of Contact impossible, so the coincidence of isoclitical crooked-lines the one upon the other makes an isoclitical angle of Contact impossible, except only in an ultradiametral positure. And as the mensuration of right lined angles is by the Arches of Circles drawn upon the angular point intercepted between the two isoclitical sides, to shew how far they are departed from their coincidence; so in isoclitical crooked lined angles, by the same way of mensuration an account may be taken of the departure, which each isoclitical side hath made from the other since their coincidence: and this is the point in

which

which their equality confifts and is accounted,
and which founds the mathematical homo-
geneity which is between them. To inftance
in the cafe which is moft manifeft ; in fig. 18.
from the angular point *A*, let the two arches
ABC. and AFH of equall circles conftitute
and contain the ifoclitical concavo-convexe
angle CAF , and let the arches ABC and
*A*FH be equall : then thorow the points C and
H draw the two right-lines AD and *A*G.
According to what is above delivered ; it is on
all hands agreed, that the ifoclitical concavo-
convexe angle C*A*H is equal to the ifoclitical
right-lined angle CAH : as is copioufly demon-
ftrable from the equall arches of Circles drawn
upon the angular point as center, cutting all
the four lines : *viz*. the *A*rches comprized be-
tween the two ifoclitical crooked-lines, are
ftill equall to the refpective arches comprized
between the two ifoclitical right-lines. For
example in the chord *A*H take any where at
pleafure the point I and from the center A draw
the arch IB cutting the arch AFH in the point
F and the right-line AEC in the point E and
the arch ABC in the point B. The arch BF be-
tween the two ifoclitical arches *A*BC and *A*FH
is ftill equall to the arch EI intercepted between
the two right-lines DA and GA. For the arches
ABC and *A*FH being equall in equall circles,
the right-lines AC and *A*H are equall : and al-

of

fo AE femidiameter is equall to _A_I femidia-
meter : and by the converfe of the fame ratio-
cination _A_B arch is equall to _A_F arch : fo as in
fhort by fuperpofition , or adaptation the arch
·BE will appear to be equal to the arch FI:
and therefore adding the common arch EF;
the arch BF intercepted between the two ifo-
clitical crooked-lines _A_BC and _A_FH is equall
to the refpective arch EI intercepted between
the two ifoclitical right-lines : and this where-
foever the point I be taken in the right-line
_A_H. So as by this common way of menfura-
tion , common to both thefe forts of angles , by
reafon of the ifocliticalnefs , and the coaptabi-
lity and coincidiblenefs of the fides in each , the
one being an ifoclitical concavo convexe angle
is copioufly demonftrated to be equal to the o-
ther being a right-lined angle. But now after
what manner are we to underftand this equality
afferted between fuch right-lined and ifocliti-
cal concavo-convexe angles? It is not an every
way abfolute equality which is between the
angles, fuch as is between two equall fquares,
or two like and equall triangles, or any two
regular and equall figures of the fame kind or,
to come nearer to the matter , it is not fuch as
is between two equall right-lined angles, or
between two equall , ifoclitical , concavo-con-
vexe angles , all whofe four fides are all of
them ifoclitical each in refpect of all the reft :

but

but as things that are like each other, are like
only in some things, and unlike it may be in
many others; such is the equality between any
two such angles; *Viz.* only a respective equa-
lity, such as is possible among heterogeneals,
and inferring a necessity of some other respe-
ctive inequalities. And such an equality may
be between two mere heterogeneals; they may
be of equal length, and different breadth, or
weight: so a Triangle, and a square and a
Circle may be all equal, either in perimetry,
or surface, but not in both; so a right-lined
angle, and an isoclitical concavo-convexe angle
may be equal in respect of the recesses which
the isoclitical sides make each from other and
from their coincidence and coaptation, but in
other respects they want not their manifest
inequalities and heterogenealness. *As* a solid to
a solid may have equal proportion that a line
to a line, yet solids and lines are heterogeneal:
so a crooked-line from a crooked-line may
make equal recesses, as a right-line from a
right-line, and yet in many other things much
heterogenealness may be in the angles which
they constitute. You will say wherein? I answer
in the rectitude and curvature of the containing
sides. *And* in these different respects two iso-
clitical concavo-convexe angles may be both
equal and unequal the one unto the other,
Viz. equal in the recesses of the sides, but une-

O   qual

qual in the curvatures of the fides: in the
fame manner as two figures may be equal
in their perimetry, or fuperficial, or folid
content, and yet be figures of different kinds
under diverfe inequalities: as the one a Rhom-
bus, the other a fquare, the one a Cylinder,
the other a Dodecaedron. So a thoufand con-
cavo-convexe ifoclitical angles may be equal
in refpect of the receffes of the fides, yet each
of a feveral kind: as a thoufand figures different
in kind may be equal in perimetry, height,
bafe, fuperficial, or folid content. But you'll
fay, what is the rectitude, or curvature of the
containing fides to the nature of angularity?
I anfwer, they are of effential concern to the
limiting and determining the nature of an-
gles: angularity being the habitude of concur-
ring lines each in refpect of the other, as to
their concurrence and inclination. And though
the inclination of ifoclitical crooked lines may
be equal to the inclination of right-lines one
upon another, in refpect of the equal receffes
and departures which the ifoclitical lines make
each from the other; yet there ftill remains a
vaft inequality, difsimilitude, and unanalo-
gablenefs between the angles and their incli-
nations, in refpect of that little of figuration,
without which neither can an angle be confti-
tuted, nor an inclination made: in a word the
fides may make equal receffes, yet be unequal

in

in their curvature, and unlike in their figuration: and neither by imagination, nor circumduction, nor any other operation can the one poſsibly be reduced, or coapted to the other, without ſetting the homologal points at improper and undue diſtances and poſitures one from another; which ſhews a ſpecifical difference between the two inclinableneſſes of the one and the other: beſides that a right-lined angle can continue its inclination between the ſides infinitely, but many iſoclitical concavo-convexe angles thereunto equal by the neceſſity of their curvature muſt terminate within a very little ſpace: circumferences and ſeveral other arches, not being poſsible to be produced beyond their integrity; ſo as ſome three given angles conſtituting a given triangle as to its angles, cannot, in like manner, conſtitute a triangle of any given magnitude; which is otherwiſe in right-lined angles. And that the equality between iſoclitical concavo-convexe and right-lined angles is not ſo abſolute as to make them every way alike, equal, and of the ſame kind, may appear eſpecially in this, which is elſewhere demonſtrated; Viz. that an ultradiametral concavo-convexe angle of Contact, being iſoclitical, is alwayes equal to two right right-lined angles, which no one right lined-angle can be: and if it be aniſoclitical of the larger, it is ever greater then two right

right-

right-lined angles can be, which is impoffible alfo for one right-lined angle to be.

And the difference between mathematical heterogeneity and the heterogeneity of angular figurations being as above difcovered ; the nature as well of anifoclitical crooked-lined and mixt-lined fecant angles will as clearly appear : *Viz.* that comparing them with right-lined angles they are compound and concrete angles conftituted of right-lined angles and angles of Contact ; which are demonftrated every way heterogeneal : and fuch anifoclitical fecant angles cannot be divided into any number at pleafure of parts homogeneal either mathematically, or in refpect of their figuration, but of neceffity fome of them muft be both wayes heterogeneal. This is manifeft, becaufe the fluxe, or circumduction of angles of Contact or of one of their conteining fides addes only a right-lined angle to them : after the fame manner as the four right right-lined angles, which compleat the fpace in any plane, about any given point, may be exhaufted by the circumduction of a crooked, concave, or convexe autoclitical, or antanaclitical line, as well as by the circumduction of a right-line. And that this ties not both angles to be of the fame kind, may eafily appear from the heterogeneity between lines and their fluxes, which are fuperficial ; or furfaces and their fluxes, which

which are folid. No wonder therefore if by
the fluxe of an angle of contact, or of one of
its fides, be created another kind of angle,
holding no analogy with the former: the he-
terogeneity and improportionablenefs of right-
lined angles and angles of Contact having been
demonftrated.

So upon the fame ground we may be affifted
to look into the fpecial properties, confide-
rable in the feveral kinds of angles peculiar
unto fome and incommunicable unto others,
for example.

In right lined angles, neither the greateft
pofsible angle, nor the leaft pofsible angle can
be given; though all ufually faid to be within
the compaffe but of one kind. But to purfue
the difference which is in angles; angles of
Contact, except fuch as are contained under
lines of the fame rectitude and curvature are
every one of a feveral kind, either mathema-
tically, or in refpect of their figuration, or
both. And except convexo-convexe angles of
Contact of equal arches, which may be di-
vided into two equal angles by common right-
lined tangents; all other angles of Contact
are, every one, both the greateft, and leaft
poffible, of their fpecial kind: and every angle
of Contact contained under the convexe fide
of its arch, or arches, is the leaft pofsible
under thofe fides; which I fuppofe was the fpe-

cula-

culation unhappily missed by those learned
men , who would have imposed upon the
world , upon that their mistake , the dream of
the coincidence of the sides in such angles.

Again a right-line may be drawn dividing a
convexe-convexe angle of Contact , whether
it divide it equally , or unequally , but a right-
line cannot be drawn dividing either of the
recto-convexe angles of Contact into which the
former was so divided ; whether the two recto-
convexe angles of Contact be of the same , or
different kinds : as is up and down demonstra-
ted in Geometry.

In citradiametral concavo-convexe angles of
Contact , either the Arches are of unequal cur-
vature , or which is tantamount , though they
be of equal curvature , yet they touch not at
homologal and answering points , being not
all over of equal curvature ; which makes them
notwithstanding the respective but not answe-
ring equality of their curvature to be anisocli-
tical. And between their Arches containing
the angle of Contact , a right-line cannot be
drawn ; but infinite crooked-lines in number
may be drawn , bearing in like manner their
convexity towards the concave which is in-
ward , and their concavity towards the con-
vexe of the other side , which is also inward.

Concavo-convexe angles concurring by way
of section , and having two right-line tangents
                                   drawn

drawn upon the Arches at the angular point
are equal unto the right-lined angles contained
under those right-lined tangents ; adding re-
spectively to each right-lined angle one of the
recto-convexe angles of Contact , and subdu-
cting out of it the other recto-convexe angle of
Contact : and when those two recto-convexe
angles of Contact are equal, as they are , when
the sides are isoclitical , then the concavo-con-
vexe angle is exactly equal to the right-lined
angle : but when the two recto-convexe angles
of Contact are unequal , as they are when the
sides of the concavo-convexe angle are anisocli-
tical then the concavo-convexe angle and the
right-lined angle are unequal.

Concavo-concave angles concurring by way
of section (as all such ever do ; or by coinci-
dence , and then one right-lined tangent gives
the analyfme of them , shewing the two recto-
convexe angles of Contact , by which the an-
gle of coincidence is less then two right right-
lined angles) are by two right-lined tangents
at the angular point reduced into the right-
lined angle ; which is the least of those right-
lined angles that are greater then it , exceeding
it only by two recto-convexe angles of Contact
to be taken out of it.

Convexo-convexe angles concurring by way
of section are by two right-lined tangents at
the angular point reduced into a right-lined
angle,

angle ; unto which to make it equal to the con-
vexo-convexe angle , are to be added two
recto-convexe angles of Contact. And the right-
lined angle is the greatest of all the right-lined
angles that are less then the convexo-convexe
angle,

Recto-concave angles concurring by way of
Contact are the greatest angles possible under
those two sides.

Recto-convexe angles concurring by way of
Contact are the least angles possible under those
two sides.

If we compare recto-concave angles of Con-
tact with right lined angles , they are less then
two right right-lined angles by one only recto-
convexe angle of Contact. And the least recto-
concave angle of Contact is greater then the
greatest right-lined angle whatsoever.

Recto-concave angles concurring by way of
section compared with right-lined angles which
are constituted , i. e. compleated by the right-
lined tangents drawn upon the Arches at the
angular points ; are less than such respective
right-lined angles by a recto-convexe angle of
Contact.

Recto-convexe angles concurring by way
of section compared with right-lined angles
which are constituted , i. e. compleated by
right-line tangents drawn upon the Arches at
the angular points ; are greater then such right-
lined

lined angles by a recto-convexe angle of Contact.

Every angle of curvature, or coincidence, having a right-line tangent drawn upon the angular point, appears to be lefs then two right right-lined angles by two recto-convexe angles of Contact.

The inclination of the fides without the angular point at any two refpective, or other points, the one taken in the one fide, the other in the other, is very nearly fhewn, and as nearly as is pofsible in right-lines, by the right-line tangents of thofe refpective points: but in mixed lined and mixed crooked-lined angles by feveral wayes of accounting, feveral points are made to anfwer one another, as by accounting by diftance from the angular point, or by accounting by equalnefs of lines along the fides &c.

Mixed lined angles of Contact, when they can be, and are, divided by a right-line, the parts are heterogeneal and unequal: and one of the unequal parts is a right-lined angle.

Every recto-convexe, and convexo-convexe, or citradiametral concavo-convexe angle of Contact is the leaft poffible under thofe fides.

Rectilineary menfurablenefs in mixt lined, crooked-lined angles concurring by way of fection, begins from the recto-convexe angle
of

ofContact : as in right-lined angles from coin-
cidence.

A convexo-convexe angle of Contact, in re-
spect of dividablenefs by right-lines is an
angle made up only of heterogeneal parts,
when it is a mixt-crooked-lined angle : but
when it is an unmixt crooked-lined angle, it
hath some parts which are homogeneal , *Viz.*
two equal recto-convexe angles of contact,
which are therein added the one unto the
other. And thofe two equal recto-convexe an-
gles of contact , as they are homogeneal,
i mean of the fame kind one with another,
both mathematically and in refpect of their fi-
guration; fo mathematically they are homo-
geneal and of the fame kind with the convexo-
convexe angle which was divided ; but in re-
fpect of it, as to their figuration, they are he-
terogeneal and of another kind.

The moft fimple angle may be divided into
heterogeneal parts : *i. e.* the inclinablenefs of the
one fide to the dividing line both in refpect of
figuration and proportion may be fpecifically
different from the inclinablenefs of the other
fide to the fame dividing line : as a pentagone
may be divided into a tragone and a triangle ,
fo a recto-convexe angle of Contact may be di-
vided into two parts heterogeneal the one to
the other, and to the firft angle of Contact,
both in refpect of figuration and proportion :

viz.

viz. into a new recto-convexe angle of Con-
tact, and a concavo-convexe angle of Contact.
Therefore no angle can be said homogeneal
in that sense, as if it could not be divided into
parts heterogeneal; whether you please to un-
derstand it, in respect of mathematical homo-
geneity, or positure, and figuration, or what
respect soever else that limits and distinguisheth
plane angles one from another

And to give a brief and general account of
the comparative admensuration of angles, as
not being right-lined, yet by way of compara-
tive admeasurement, they may in respect of
their rectilineary parts be reduced and referred
to those that are right-lined; the containing
sides not being right-lines, at the angular point
draw right-line tangents touching the arch, or
arches in the angular point; and the right-lined
angle contained by those right-lined tangents
will be, as to the recesses of the sides at the an-
gular point, either equall unto the first propo-
sed angle, or the least right-lined angle greater
then it, or the greatest right-lined angle lesser
then it: or if two right-lined tangents cannot
be thus placed at the angular point, either the
first proposed angle was a mixt-lined angle of
contact, said, if a recto convexe to be less,
if a recto-concave to be greater then any right-
lined angle; or else it is a crooked-lined angle
of contact, which if convexo-convexe, or con-
ca-

cavo-convexe and citradiametral is lefs then
any right-lined angle , but if concavo-convexe
and ultradiametral, is greater then any right-
lined angle, nay fometimes equall to , or grea-
ter then two right right-lined angles : or elfe it
is an angle of coincidence, or curvature. All
which is to be underftood to fhew the inclina-
tion of the fides at the angular point, as the
chief for ufe in Geometry, but not neceffarily
elfe-where. So crooked-lined, or mixed lined
angles are compared with right-lined angles by
drawing at the angular point right-lines touch-
ing the Arches there, and comparing the
crooked or mixed lined angle with the right-
lined angle fo conftituted , refpectively adding,
or fubducting the recto-convexe angles of Con-
tact hereby created: and this, whether the
Arches be ifoclitical , or anifoclitical , or how-
ever pofited. So all crooked , or mixed-lined
angles concurring by way of fection may have
a right-lined angle given , which if it fall fhort
of equality is either the leaft of the right-lined
angles that are greater , or the greateft of the
right-lined angles that are lefs then the firft
crooked-lined , or mixed lined angle. And fo
an analyfme may be made of the greater angle
into its heterogeneal parts; and the crooked
lined, or mixed-lined angle may be reduced
unto, or compared with right-lined angles ,
only with the addition , or fubftraction of recto-

con-

convexe angles of Contact, being angles lefs than the leaft right-lined angle whatfoever.

All angles have their inclinations compounded of the inclinations of the interjacent lines each to other in order : and of the inclinations of the fides to the lines next adjacent to them ; which compofite inclination may be heterogeneal , as well as homogeneal in refpect of the inclinations of which it is , or may be compounded.

Equally arched convexo-convexe , or concavo-concave angles , may by a right-line be divided into equal parts mathematically homogeneal , but heterogeneal in refpect of their figuration : but fuch angles cannot be divided into any more , or any other equal parts , for the reafon immediately to be fubjoyned.

Heterogeneals taken together in feveral concretes proportionably , *Sc.* each refpectively in the fame proportion, they hold exact proportion , concrete to concrete : as double cube and douUe line , are double, to fingle cube and fingle line , *Viz.* the concrete to the concrete : but fet them out of the fame refpective proportions and the concretes are no way proportionable , or in analogy , concretely to be compared : as double cube and treble line , are in no proportion, to fingle cube, and fingle line. So double number, and double weight , and double meafure, the whole concrete, is

dou-

double , to fingle number , fingle weight and
fingle meafure : but fetting them out of the fame
refpective proportions ; double number , and
double weight and treble meafure , being all-
together concretely taken , are mathematically
heterogeneal aud improportionable to fingle
number , fingle weight and fingle meafure ;
being in like manner concretely taken : becaufe
the heterogeneals in the one concrete hold not
the fame refpective proportions to the anfwe-
ring heterogeneals in the other. So convexo-
convexe , or concavo-concave equally arched
angles being fecant , hold proportion when
divided equally , as they may , by right-lines :
but they are merely heterogeneal and without
proportion , when divided by a right-line une-
qually. The ground of which is the hetero-
geneity of the parts , of which fuch concrete
angles are made up when compared with an-
gularity conftituted by right-lines ; which he-
terogeneal parts , when the angle is divided
equally in two by a right-line , are in the con-
cretes , each refpectively in the fame propor-
tion ; fo making the concretes , though of he-
terogeneal parts , to be mathematically homo-
geneal and proportionable one unto the other :
but when the angle is unequally divided ; in
the two concretes the heterogeneal parts , of
which they are made up , are not refpectively
in the fame proportion ; for the recto-convexe
<div align="right">an-</div>

angles of Contact, in the concrete angle, are divided equally, and the right-lined angle, which is in the concrete angle is divided unequally: fo making the parts of the divided angle mathematically heterogeneal and improportionable ; because the compounding heterogeneal parts are not refpectively according to the fame proportion divided.

The leaft p ffible angle under any two given incoincidible lines is the leaft angle of Contact which is poffible under them.

Recto-convexe, and citradiametral concavo-convexe angles of Contact as they are the leaft angles poffible under their fides ; fo they are indivifible into parts holding all each to other any thing of mathematical homogeneity, or proportionablenefs : Like unites they may be multiplyed to any proportion as whole numbers, and feparately fet : but cannot at pleafure be divided, nor at all into parts which are all of them mathematically homogeneal : nor can any number of them be at pleafure adjoyned one to another. So to any right-lined angle another may be imagined in any proportion, but fometimes it muft be the compofition of fefeveral angles, and more then can ftand at the fpace circumjacent about any one point. It not being pofsible for above four right rightlined angles to ftand about the fame point. As to a given point, other points may be given in
any

any proportion , as whole numbers ; but they muſt not then be adjoyned one to another : but properly by the dividing of a point , farther proportions are not to be expected. So in ſuch angles , though heterogeneal diviſions be infinitely poſsible ; homogeneal, I mean mathematically , being impoſsible , are not, as above , to be expected.

Two unequal aniſoclitical angles contained under the ſame two aniſoclitical ſides , though in reſpect of their figuration , both are formed upon the ſame inclinableneſs of the ſides one to another , yet they are mathematically heterogeneal : becauſe being two concrete angles made up of heterogeneals , _viz._ of the angle , or angles of Contact and right-lined ! angles ; in the two concrete angles , the angles of Contact are in the proportion of equality , and the right-lined angles in the proportion of inequality : ſo as the concrete angles can have no proportion the one to the other.

The half of the inclinableneſs of any arch pon its ſelf, _i. e._ upon another arch like and equal, is ſtill comprehended and contained under the inclinableneſs of a right-line upon the ſame arch : that though their figurations are ever heterogeneal , and their inclinations can never be equal , yet they may be mathematically homogeneal : as different numbers are alwayes unequal though ever proportionable.

To

To no anifoclitical angle, whether of Contact, or concurring by way of section can any right-lined angle be made equal. And generally between ifoclitical and anifoclitical angles equality is impoffible. And when anifoclitical angles are compared with thofe that are ifoclitical as greater, or leffer; it is not to denote in both any mathematical homogeneity, and that fo by poffibility they are reducible to a true, compleat and analogous equality; but the intent is only to declare, whether of them hath, or can have the containing fide, or fides falling within, or without the other; which is only from their inequality by the whole kind. So though anifoclitical angles may be greater, or lefs then a right-lined angle, yet between the one and the other there is neither common way of meafuring, nor any proportion, though both of them be quantitative plane angles. There is no common way of meafuring them, becaufe of the anifoclitical-nefs, and if there could be any proportion between them, then might a right-lined angle be given equal to fuch anifoclitical angles; the contrary of which is otherwife clearly demonftrable. So as a thoufand angles quantitative by confeffion and having right-lined angles leffer and greater then themfelves, yet can have no right-lined angle equal to them.

A right-lined angle cannot have its inclina-

tion

tion at the angular point, much lefs all along the fides divided equally by a crooked-line, whether autoclitical, or antanaclitical, or of what curvature foever : though any fuch crooked-line may from any angle divide equally the plane bounded within a right-lined triangle: for if in fig. 17. BAC be a right-lined angle : I fay, no crooked-line &c. can divide it at the angular point equally. If it be poffible let it be divided into equal parts by the arch AGE whether autoclitical, as when it is conceived to be part of the arch AGEF, or antanaclitical, as when it is conceived to be part of the arch AGEHD. Then let the right-line AD divide equally the right-lined angle BAC. It is manifeft, if the crooked-line AG never occurre with the right-line AD, that then it doth not divide the angle BAC equally ; for the right and crooked-lines cannot be coapted. Let therefore, if they do occurre, the place of their firft meeting, or occurrence be at E. Therefore the arch AGE falleth within the angle BAD and therefore divides the whole angle BAC unequally, whether it be autoclitical, or antanaclitical. And by the fame demonftration appears that it is as impofsible for it to divide the inclination all along the fides equally. And though a crooked-line may divide any right-lined figure, becaufe of its perfect bounds, from any angle into

into equal parts ; yet this no way evinceth any posſibility of dividing a right-lined angle by a crooked line into equal parts : and though from any point of ſuch a dividing crooked line, lines might be drawn making up a figure, whoſe parts divided by the crooked line from the angle are equal ; yet this is no more then is performable upon any point of any crooked line drawn between the ſides of a right-lined angle at randome, whether the parts of the angle be homogeneal, or heterogeneal, equal, or unequal.

Difference of curvature by a perpetual neceſſity infers difference of inclination, whether the curve line be inclined upon a right-line, or upon a curve line ; for ſtill the one of the curve lines will fall within, or without the other. So let inclination upon a right-line be in a recto-concave angle of Contact, recto-convexe angle of Contact, recto-concave, or recto-convexe ſecant angles ; by no right-line can any of thoſe inclinations be made upon the firſt right-line, nor by any other crooked-line : but ſtill the lines will fall either within, or without. And all angles of Contact under lines of different curvature and rectitude, as they are of a thouſand Mathematically and extra-quantitatively different kinds one from another, ſo they are as manifeſtly diſtinct in kind from all right-lined angles wha ſoever.

And as every number is a different kind, in like manner as to positure and figuration is every line a distinct kind differing from all other not agreeing with it in rectitude and curvature; whether the curvatures be homogeneal, *i. e.* every where equal, as in Circles, or heterogeneal, *i. e.* unequal in the several parts, as in ellipses, hyperbolas, parabolas, &c. And accordingly judgment is to be made of the sides under which angles are contain'd, and the angles contain'd under them: alter the kind of either side, and the kind of the angle is chang'd; ever, in respect of figuration; and most what, mathematically; because the former inclinablenes of the sides is taken away, and a new kind of inclinablenes introduced between them. Hence appears; no two recto-concaves, no two recto-convexes, no two concavo-convexes, no two convexo-convexes, being all angles of Contact can be equal, except their sides have the very same rectitude and curvature. In general, equality is not to be asserted between angles, except either for the menfurablenes of both in some common way of measuring, or at least because they can be so cast into a coaptation, as that either shall contain all the quantity which is in the other. And as he erred in squaring the superficial content of a circle, that in his quadrature left out a small lunular figure; so neither can he be justi-

juſtifyed to have given one aniſoclitical angle
equal to another , who , as he muſt of neceſſi-
ty, takes in , or leaves out an angle , or ſome
angles of contact which are in the one and
not in the other.

Particularly no mixed lined , nor mixed
crooked lined angles whatſoever , whether
ſecant, or of contact , can by any poſſibility ,
either by right-line , or crooked line be divided
into equal parts , or angles, whether of the
ſame , or of diſferent, kinds. And in few , thoſe
angles , which cannot be divided into two
equal parts cannot be divided into three ,
fower, five , or any other number of equal
parts.

The comparative admenſurement, as above,
of right-lined and not right-lined angles is as
well of their heterogeneal inequality, when he-
terogeneal, as of their homogeneal equality and
proportionablenes when homogeneal.

All not right-lined angles comparatively
admeaſured , to right-lined angles , only with
the addition, or only with the ſubduction cf
any angle , or angles of contact , are hetero-
geneal to all right-lined angles , as alſo , if
when one angle of contact is to be added , and
another ſubducted , the two angles of contact
be unequal. And as is manifeſt , of ſeveral not
right-lined angles , that is ſtill the greater ,
whoſe comparative and relative admeaſure-

ment

ment is made to the greater right-lined angle:
and of thofe, whofe comparative admeafurement
is unto equal right-lined angles, they are grea-
ter, or equal, or lefs according to the equality,
or inequality, and comparing together of their
angles of Contact. And of concavo-concave
angles the right-lined angle cannot be given,
which is the greateft of thofe right-lined an-
gles that are lefs then the concavo-concave
angle: nor in convexo-convexe angles can be
given a right-lined angle, which is the leaft
of thofe right-lined angles which are greater
then the convexo-convexe angle: nor in recto-
concaves the greateft of the lefs: nor in recto-
convexes the leaft of the greater: whereas in
concavo-convexe angles, the right-lined angle
to which the comparative admeafurement is
made, may according to the cafe be either
equal, as when the fides are ifoclitical; or the
greateft right-lined angle that is lefs, as when
the concave fide is of lefs curvature; or the leaft
of the greater, as when the concave fide is of
greater curvature.

From thefe things may appear how that
objection is to be anfwered, in which you
urge that there is a proportion between mixed
lined angles, whofe fides are fecant, and right-
lined angles, becaufe the one by its multiple
may exceed the other; and that therefore an
homogeneity, mathematically to be under-
ftood,

ftood, is to be acknowledged amongft them.
To this is anfwered that all anifoclitical angles
whatfoever having their fides concurring by
way of fection when compared unto right-
lined angles are heterogeneal compofitions of
angularity, *i.e.* cannot be divided into any
number at pleafure of parts all equal ; but as is
manifeft, they in that their relative nature are
concrete and compofite angles , formed of
right-lined angles by adding to them , cr fub-
ducting from them mixed lined angles of Con-
tact: fo as the right-lined part of the fecant
anifoclitical angle , or any the leaft part of
it hath true proportion with all right-lined
angles ; and the whole being heterogeneal ,
yet hath a femblance of proportion with right-
lined angles , becaufe of thofe parts which it
hath which are homogeneal with them ; but
between the other heterogeneal part which is
the mixt-lined angle of Contact, and right-lined
angles , neither is , nor can be any proportion :
and it is becaufe of this part that it is not a true,
but a feeming proportion only , which is be-
tween the whole concrete heterogeneal , com-
pofite anifoclitical angle , and right-lined an-
gles: for if it were a true proportion and pro-
portionablenes which were between them , it
fhould then be poffible to give a right-lined
angle equal to fuch an anifoclitical fecant
angle ; which when done , I have no more to

fay,

say, being well assured I am able to demon-
strate the contrary upon whatsoever right-lined
angle shall be offered under that notion. And
whereas it is urged that a right-lined angle
is, and may be equal to a mixed lined angle;
all their difference being only in this, that the
sides of the one are more spread and divari-
cated then the other; what is this else but
to say, that they are equal, saving that the
one is bigger then the other. For angles of
Contact can divide angularity, or space equal-
ly, or unequally into more, or fewer parts:
and they are not indivisible, because every one
contains innumerably more in it: and by the
definition of an angle, lines have a sufficient
inclination to constitute an angle, if in the same
plane, they lye not both in the same right-
line. And to all other cases and objections
of the like nature grounded upon them athema-
tical heterogeneity of the parts, of which such
special angles are constituted in respect of right-
lined angles, like answers may be addressed.

Though a right-lined angle cannot be di-
vided into equal parts of the same kind mathe-
matically, or in shape by a crooked-line, nor
a mixed, or crooked-line angle by a right-
line into parts of the same kind in respect of
figuration; yet some crooked lined angles may
be divided into parts exactly equal of the same
kind one with another, and with the whole
ma-

mathematically by a right-line. For it is ap-
parent, there may be between magnitudes a
fufficient homogeneity for proportionablenes
without excluding all further difcriminablenefs
between them : as between equal arched con-
vexo-convexes , or concavo-concaves and their
parts when they are equally divided. So be-
tween lines of all kinds , and numbers of all
kinds , is proportionality. Yet ftill their propor-
tionality is in refpect of fomewhat which is
homogeneal in them. *Viz.* that they are all re-
foluble into parts that are homogeneal , or
parts into which the reft are homogeneally
refoluble , or after the fame manner menfura-
ble , as equal arched convexo-convexe angles
into two equal recto-convexes : yet notwith-
ftanding the proportionablenes between the
convexo-convexes and the recto-convexes ; fuch
an heterogeneity is in their figuration , that by
no divarication of fides can they ever be made
equal *A*nd though the equal arched convexo-
convexes and concavo concaves may be divi-
ded by a right-line into two equal parts ; by
no lines whatfoever can they be divided into
any more equal parts nor their recto-convexes
by any line whatfoever into two equal parts ;
fuch parts of fuch angles being more impof-
fible to be given in Geometry , then the fquare
roots of unfquare numbers, or cubicke roots
of uncubical numbers are in *A*rithmeticke. As
any

any two points, or any two indivifibles may
have fome few proportions amongft them , but
no more: the latitude of proportionablenes
being limited by the fpecial natures and kinds
of things: fo between fpecial kinds of numbers
cannot be all proportions: nor between fpecial
lines all angles, or all proportions in all an-
gles.

Hence alfo we may clear the poffibility of
that fpeculation of the learned perfons by you
named , that in heterogeneals there may be a
paffing from greater to lefs , and from lefs to
greater in a continuous manner without , ever
paffing thorow equality ; however you are
pleafed to beftow upon it a contemptuons fmile,
if not an hifs : *Viz.* that we fhould affert that
mixed lined , or crooked lined angles can
pafs by the divarication of the fame fides from
being greater to be lefs then given angles , or
contrarily by their contraction , and yet in that
tranfit , never be equal. What is more obvious
then to give inftances of anifoclitical angles,
lefs then a given ifoclitical angle, and by di-
varicating and diftending the fides of the ani-
foclitical angle , it may be made to exceed the
firft given ifoclitical angle; yet in all the way
they could never be equal the one to the other;
the one being ifoclitical, the other anifocli-
tical: coaptation and the paffing of the fides
from the angular point every where prove
                                        their

their inequality : and let thofe that aſſert their
equality any way ſhew and admeaſure it. And
the ſober underſtanding of thoſe ſayings, that
ſuch angles paſſe from leſs to greater without
ever being equal , is not to aſſert any homo-
geneity, or proportion of any kind between
them, or common way of meaſuring their quan-
tity in and according to the ſame indefinite
quantity, but only to ſhew how the ſides may
paſs within and without each other; but be-
cauſe of their aniſocliticalnes and difference in
reſpect of rectitude and curvatures, they can
never be brought to be coincident. And why
ſhould this appear ſo monſtrous , that tranſi-
tions ſhould be , in the ſenſe aboveſaid , made
from greater to leſs , without paſſing thorow
equality? ſeeing it is moſt manifeſt that though
a crooked-line circumducted about the angular
point of a right-lined angle makes infinite
diviſions of the right-lined angle ; yet they are
ever unequal , and never by poſſibility can be
equal : though notwithſtanding , the ſame
crooked line may divide a crooked lined iſo-
clitical angle eqnal to the given right-lined
angle into equal parts ; which beſides plenti-
fully ſhews the heterogeneity which is be-
tween equal right-lined and crooked-lined iſo-
clitical angles , notwithſtanding their equa-
lity: yet by making up the right-lined angle
into a compleately bounded figure , the croo-
ked-

ked-line may divide the plane of it into equal
ifoepipedal and iforrhopical parts: as is more
eafy to demonſtrate then that there ſhould be
any need to ſet it down. And it is not to be
ſtranged at, that we aſſert a crooked-line can
divide a right-lined triangle into equal parts,
but not any of its right-lined angles: for the
figuration of the one is compleat, and the
production of the crooked-line, as well as of
the lines containing the right-lined angle, in
the triangle are limited; all which are quite
otherwiſe in mere angles, being in many
kinds of angles and inclinations very alterable
by the production of the lines. And if you will
pertinaciouſly ſay that a crooked-line may di-
vide equally a right-lined angle, ſhew their
equality, and your way of admeaſuring the
equality of the parts: both coaptation of ſides,
in whoſe habitude the nature of angles chiefly
conſiſts, and alſo the way of meaſuring by
interjected arches demonſtrating and declaring
the contrary: Nay ſometimes in quantities
among which there is true proportion, the
ſpecial differences of their kinds may be the
authour of little leſs. So commenſurable quan-
tities, being in their commenſurability infi-
nitely diviſible, may be continually increaſed,
or leſſened by quantities vaſtly, leſs then any
given quantity; yet as they paſs from leſs to
greater never can hit equality with any the like
quan-

quantities of the same general kind, being incommensurable. So in the present question, heterogeneity of sides makes an impossibility to coapt, though by straitning and divaricating they may fall within, or without, and so be less, or greater, but being impossible to coapt, and incapable of any other common way of measuring, equality cannot be conceived in them; in which is præsuppofed a common way of measuring to declare and prove their afferted proportion and equality. And in this whole matter what is more said then that the cutting inclination of a right-line upon a right-line, and of a right-line upon a crooked, or of a crooked-line upon two crooked-lines being all of different curvature, may any of them be greater, or less then one another heterogeneally, but can never be equal; as to those that will confider, is most clearly manifest and necessary. And this is no more strange, then that odde numbers may be less, or greater then any even number, at least above two, yet by their constitutive nature, they can never be equal: and for the same reason magnitudes commensurable and incommensurable may be greater, and less then one another, and by a less quantity then any quantity that can be given; yet, for the specialty of their natures, they can never be equal, remaining so distinct in their ultimate kind: and however they have

many

many general things in which they agree , yet they are not accountable fpecially , but by feveral wayes and algorythmes. And as it is the binding up of numbers to the fpecifick properties of Evennefs , or oddnefs , and of magnitudes to commenfurability , or incommenfurability , that makes equality between their kinds impoffible ; fo it is the incoaptablenes and want and impoffibility of a common way of meafuring , which is between the two and two fides of fuch heterogeneally unequal angles , keeping all along the properties of their inclination , that renders equality between their kinds impoffible ; though either may fometimes truly and manifeftly be greater or lefs then the other , however without proportion. For in heterogeneals may be a two fold inequality , either an inequality according to proportion , or an inequality without proportion : and though they be heterogeneals , if it be a true limited , determinate proportion of inequality which is between them , as between an odde number and an even , a commenfurable magnitude and an incommenfurable , they are then confidered as fome way , or according to fomething which is , homogeneal , not heterogeneal , in both : but if their inequality , though real and apparent be without any true and homogeneally determinable proportionablenes , as between right-lined and

<div align="right">anifo-</div>

anifoclitical angles, then the confideration
had of them is in their heterogeneity. So to the
objection, that right-lined angles, and recto-
convexe angles of Contact have proportion of
greater and lefs, and are therefore homogeneal,
if any quantitativenes be to be afferted in the
recto-convexe angles of Contact; and fo of the
reft : I anfwer, there is not a proportionable
inequality between right-lined angles and recto-
convexe angles of Contact, and the like ; but
an inequality, which is improportionable:
as when the Earth is faid to be intruth greter
then a point; though in many particular hy-
pothefes contrarily conceived, for the better
obferving and accounting feveral phænomena,
and the better accommodating of inftruments
for the making of obfervations. So in the ge-
nefis of quantities and figures, when one quan-
tity hath its genefis by the fluxe, or motion
of another, the quantity formed by that motion
is greater then the quantity moved, though
ftill without any proportion: fo by the diva-
ricating of the fides, by the motion of one of
the fides, of a recto-convexe angle of Contact,
is a genefis of recto-convexe angles of fection ;
and the recto-convexe angles of fection are ftill
greater then the recto-convexe angle of Con-
tact, or its refpectively moved fide : and all
ftill without any proportion. And inequality
without proportionality being fo ufually affer-
ted,

ted, and familiar between mere heterogeneals, doth easily acquit it self from the reproach you charge it withal, of being a contradiction in the very terms; for finite and infinite never were denyed to be unequal, yet never can be made out to be mathematically homogeneal and proportionable. For though all proportion is either of equality, or inequality; that hinders not but an inequality in heterogeneals may be admitted, without an asserting of proportion between them. So as mathematical homogeneity is not proved by inequality, till proportionablenes be as well proved as inequality. Of the same leaven is that strange kind of reasoning you use; to any right-lined angle a crooked-lined angle may be made equal (we confess a thousand several crooked-lined angles may be made equal to any one right-lined) angle) but you from thence inferre that therefore all crooked lined angles and right-lined angles are of the same kind; without adding as well, that to any crooked-lined, or mixed lined angle, a right-lined angle also might be made equal; which can never be done; the contrary thereof being confessedly demonstrated in Geometry: or if any be so opinionated that it is either easy, or feasible, let them give a right-lined angle equal to an anisoclitical angle, whose sides concurre by way of section or to an ultradiametral convexo-concave angle

of

of Contact : and trye whether the equality of the two angles be not difproveable. In the fame manner inequality without proportion, is afferted between both the recto-concave and recto-convexe angles of the femi-circle, *i. e.* the inner and outer angles of the femi-circle, and a right, or any other right-lined angle. But to return, from the concrete and compofite nature of not right-lined angles, when compared with right-lined angles; the enigmaticalnes of the propofition, that magnitudes of one fort may be greater, and lefs then a given magnitude of another, but never equal, becomes moft clear and doubtles : as infinite folids may be given greater, or lefferthen the heterogeneal concrete magnitude of a cube foot and a foot-line; but never can any folid be given thereunto equal : fo likewife infinite heterogeneal concrete magnitudes confifting each of folids with the accrefcency, or annexion of a foot-line, may be given greater, or leffer then a cube elle ; but in fuch comparifons, never can the concretes of the heterogeneal magnitudes be equal, or in proportion to any folitary, fingle, one, of the heterogeneals.

In like manner it manifeftly appears what is to be thought of that often pretended equality between the two angles of a femi-circle and a right right-lined angle : for by coaptation and
the

the goings forth of the lines from the angular point it appears otherwife. *And* how will they that affert their equality demonftrate it? and by what way will they admeafure it? for interjacent and intercepted arches of Circles drawn upon the angular point as center, can in thefe angles contribute nothing to the admeafuring of them: and coaptation makes againft their equality: and is fo farre from making a right right-lined angle to be the conftant ftandard of all angles of femi-circles, that it manifeftly fhews the angles of greater femi-circles to be greater and of lefs, leffer. The objection, that unequal circles, femi circles, and fegments of equal degrees, cannot be judged like and homologal figures, except all their refpective angles be equal, as well as the fides homologal: this may be eafily anfwered; *Viz.* that it is like genefis that makes like figures, which in all right-lined figures makes the anfwering angles equal, and in all figures the fides to be homologal, and the difference of the homologal angles to be lefs then the leaft right-lined angle.

In Circles the fame genefis, by the circumduction of the femi-diameter about the Center, which makes the figures like, and the fides homologal, makes in unequal circles the curvatures, angles of coincidence, the angles of the femi circles, and of homologal fegments, necef-

neceſſarily unequal. Or, when figures are to
be judged like when in a concentricke po-
the perimeter, ſides and lines of the on-
proportionable to thoſe in the other, an
or may be placed parallel, or coincid
thoſe in the one, to thoſe in the other ;
which in right-lined figures, tis true, there
alwayes follows equality between the anſwer-
ring angles, but not ſo in other lined figures.
And becauſe equality of angles in like right-
lined figures is ſo much urged ; the differ
of the caſe of angles in like right-lined and like
crooked-lined figures may plainly appear in
another remarke ; *Viz.* that in like right-lined
figures, the ſides of the anſwering angles may
from the anſwering angular point be exactly
coapted one to another ; which in unequal
Circles, though never ſo like figures, is moſt
apparently impoſſible : that it cannot but be
unreaſonable to expect as abſolute a conformity
between the anſwering angles in like crooked
lined figures, as there is in like right-lined
figures ; being ſo manifeſtly againſt both the
eye and demonſtration. And therefore like
ſegments of Circles, are not defined by equa-
lity of angles contained between their arches
and their chords ; but by the equality of the
angles contained in them ; the one being cer-
tain, conſtant in all, and demonſtrable ; the
other in moſt caſes, not only doubtful, but

impoffible. So the objection, from the afferted
equality of alternate angles, made by a right-
line cutting parallel circumferences; is readily
anfwered by denying the truth of what is pre-
fumed in the objection, as never by any de-
monftrated, or ever poffibly demonftrable,
Viz. that the alternate and vertically oppofite
angles, made by a right-line cutting parallel
circnmferences are eqnal. And the tyranny of
forcing lines out of their natures and fpecial
properties, may appear in that very inftance
of compelling the parallelifme of curve lines
to anfwer the confectaries and idioms of the
parallelifme of right-lines: in which, to omit
the alleadged inftance, as by you unproved,
and for good reafons by us to be denyed, a
right-line tangent of the leffer concentrick
Circle cuts the circumference of the greater,
and infinite right-lines cutting the greater,
neither cut, nor touch the leffer, which is
repugnant to the nature of parallelifme in right
lines. That that which is fo much contended
for, that a crooked-line aud a right-line are
homogeneal as to length and their general
linearinefs, was, or ought never to be denied,
there being all poffibility of equality, and
truly proportionable inequality between them,
what kind of curvature foever the crooked-
lines bear: but that they are homogeneal as
to the pofiture of their longitude; the fite and

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manner of their extenfion , hath unto me
been alwayes unconceivable : whence the
truths on both hands clearly follow , *viz*.
that an arch and a right-line may be equal and
hold alwayes a true limited , exact proportion
one to another , but the arch and its chorde
never can be equal , *i. e.* there never can be
equality between a right-line and a crooked-
line , both pofited between the fame two ter-
minateing points , nor any analogy between
the rectitude of the one and the curvature of
the other. And the feeking to prove the equa-
lity of angles contain'd under homologal lines ,
the one curve , the other a right-line from your
ufual fancy of a regular polygone of infinite
angles in every circle , is too wild to be per-
fwafive : for though at any mean point in
curve lines , the two parts of the curve-line
may be conceived fpecially to meet as feveral
parts and lines , and fo to have inclination the
one to the other , and fo to conftitute an angle;
which we call the angle of curvature and co-
incidence , not reafonably to be denyed
by thofe , with whom it is fo ordinary to
make fuch fuppofitions , and efpecially fuch as
can fo ufually againft poffibility imagine angles
in a right-line remaining a right-line : yet that
angles fhould be without fides , and a peri-
meter of any figure conceived at once to be all
angular points and no lineary fides , clearly

de-

deveftes the perimeter of the nature of a line: and to me it seems far from the nature of a regular figure, that hath nothing but points inftead of lines to bound it : but which is moft material; that a number actually infinite fhould be fo eafily given, is hard to allow : and that indivifibles, as points; fhould be fo adjacent one to another, one without another, without coincidence, identity and unity, is new Philofophy, and not eafily capable of any ingenious defenfe. Therefore that argumentation of yours, that fuch a regular polygone of infinite whether fides or angles is either a circle, or infcribable in a circle, is too vain: for it can be neither, being nothing, becaufe there neither is, nor can be any fuch thing: for if any fuch were allowed they muft of neceffity have equal and infinite perimeters ; which is too grofs to be admitted in it felf, and befides renders the whole matter unapplyable to Circles, which are acknowledged, to be fome lefs then others. So as all difcourfes of a regular polygone of infinite angles, are difcourfes not only of a non-entity, but an abfolute impoffibility; which renders all fuppofitions thereof unjuftifiable. And of the fame finenefs are thofe fayings, that the magnitude of an angle is not to be judged of from the divarication which the fides have without the angular point, or point of concurrence, but from the
diva-

rication which they have in the point of con-
currence ; as if in an indivisible point they
could have any divarication at all. But as if it
were refolved that even this fhould be tranfcen-
ded in monftrofity , for the juftifying of the
equality of mixed lined angles contain'd by
homologal fides in unequal Circles , by an in-
ftance from the coapting of unequal hexagones
to the fame line , as a common fide in them all,
divided equally by a perpendicular paffing
thorow the centers of all , a right-lined angle
is ftrangely conftituted either of three right-
lines concurring , but not in the fame point,
or of two lines without any concurrence, cr elfe
the inftance muft be void of all pertinency to
the queftion. So to all thofe objections feem-
ingly founded upon that propofition , or po-
ftulate , that what is lefs then any pofitive
quantity whatfoever is not any quantity at all,
is juftly anfwered ; that the propofition or
poftulate is moft true and reafonable , and can-
not by any of found mind be denyed , or doub-
ted: but no force of objection could be made
out of that , if other things of a lefs veritable
nature had not been taken in ; as in moft of
them the fancyed poffibility of a regular poly-
gone of infinite angles ; and frequently that a
Circle is that regular polygone.

But befides , though what is lefs then any
pofitive quantity whatfoever be not any quan-

tity

tity at all ; yet this hinders not , but quanti-
tyes may be mathematically heterogeneal and
improportionable one to another : fo every
furface is lefs then any folid : and angles of
Contaƈt are not lefs then any quantity what-
foever, for there is in the leaſt of them an
endles, unexhauſted divifibility ; which how
it can confiſt with a nonquantitativenes, let
thofe that have a mind to be ſerious folemnely
confider. To the objeƈtion that would prove,
neither femi-circumference to contain an angle
with the right-lined tangeut of it in its extreem
point, becaufe the two femi circumferences
contain no angle at that point, but are one re-
gularly continued line , and the circumference
and right-lined tangent are lines coincident ,
at leaſt as to the point of Contaƈt ; manifeſt and
reafonable anfwers cannot be to feek out of
what hath already been faid. For firſt what
hinders the reafonable conceiving of angularity
at any point of a curve-line , where is both con-
currence, inclination and divifibility , more
then the notion of divifibility at any mean
point of a right-line ? And not to doubt but a
curve line may be conceived reafonably as one
continued line , as well as two , or more , in-
clined and concurring right-lines ; yet that the
right-line tangent and curve-line which it
toucheth , ſhould be faid to be coincident
lines, in fuch fenfe as to exclude angularity;

or

or that any two liues can be so coincident in one only point , as to exclude angularity , and the inflexion of one to , or from the other , except both lye in one and the same right-line; hath , as elsewhere , been plainly and abundantly answered to. To the objection that the Area of a Circle is equal to a rect-angle under the semidiameter and semi circumference ; and that therefore the semidiameter in a Circle is perpendicular to the circumference in a Circle and makes at the circumference four equal right-angles ; is answered , that the whole objection is a manifest paralogisme. For it is not denyed but in the right-lined rect-angle under the semidiameter, and a right-line equal to the semi circumference , is presumed , and by the definition of a rect-angle inferred , that theangle under those two right-lines is a right right-lined angle : besides it is not denyed , but the diameter falls perpendicularly in the circle upon the circumference : and that the four angles made by the falling of the semidiameter upon the circumference differ from one another less than the least right-lined angle : however that cannot force the falling of the semidiameter perpendicularly upon the circumference into the properties of perpendicularnels between right lines, which still divides the space at the angular point into four angles always, every way alike, and equal ; which in right lines perpen-

pen-

pendiculars upon curve lines in the fame plane,
is impofsible to be , and therefore impofsible e-
ver to be demonftrated.

It will not be unufeful here to enquire where-
in the likenefs and unlikenefs of angles doth
confift , and whether there be any fuch thing as
likenefs and unlikenefs in angles , or whether
the likenefs, or unlikenefs of figures be only in
the fimilitude, or diffimilitude of the fides. And
that by a circumfpect confideration of the nature
of mathematical fimilitude in other cafes , we
may be the better guided into the true and moft
rational notion of fimilitude in angles , let us
remember what hath already been judged in
this point , and what is herein confeffed on all
hands. Firft, in right lined figures, thofe fi-
gures are judged like ; whofe anfwering angles
are equal , and the anfwering fides and other
lines proportionable ; and if they be equal, they
may be coapted , homologal fide to homologal
fide , and anfwering angle to anfwering angle ;
or whether they be equal, or unequal , all the
fides and other anfwering lines of the one may
be fet , as from the fame center , each at paral-
lelifme , or coincidency with the anfwering
fides, or lines of the other , fo as in like right
lined figures is proportionablenefs in the anfwe-
ring fides , equality of the anfwering angles,
coaptability of all the anfwering fides into ei-
ther coincidence, or equidiftance , and a pro-
por-

portionate diſtance of the anſwering angles,
each from the other. But now the ſimilitude
of figuration which is in circles founded upon
the like geneſis of all circles is in the equidi-
ſtance, or coincidence of their circumferences,
when the center of the one is coapted to the cen-
ter of the other; and that equal angles from the
center intercept proportionable parts of the cir-
cumferences, and that proportionable parts of
the circumferences are connected by proportio-
nable chords, and contain and ſuſtain equal
right-lined angles. And the like ſpeculations
might be purſued in other figures both plane
and ſolid. In a rational application of which
to the diſquiſition of angles, it may be firſt en-
quired, whether there be any ſuch thing as ſi-
militude & diſſimilitude to be own'd or obſerv'd
among angles; and if ſo, how that ſimilitude is
to be underſtood; and whether it be inconfi-
ſtent with inequality in the anſwering angles.
To clear all which we muſt know, that in une-
qual but like right lined figures, the homolo-
gal angles are always equal, being contained
in both figures under right lines: but in une-
qual, and like mixed lined and crooked lined fi-
gures, the homologal mixed lined, or crooked
lined angles, neither are, nor can be equal;
only their difference is ever leſs than the leaſt
right lined angle; and their ſimilitude hath ne-
ver rationally yet by any been queſtioned but
with

with good reason according to the following
gloſs is to be juſtified. The more clearly to
demonſtrate all which in Fig. 21. upon the
common center A. draw two unequal Circles,
viz. HEG. the leſſer, and BCD. the greater.
Then from any point B. in the greater circle
BCD. draw the right line BEAFK. thorough
the common center A. cutting the circumfe-
rence of the leſſer circle HEG. in the point E.
then take AF. equal to BE. and upon the cen-
ter F. and ſemidiameter FE. draw another cir-
cle ECKD. equal to the greater circle C B D.
Here on all hands is agreed, that the leſſer cir-
cle HEG. and the greater circle CBD. are like
figures, and that therefore the two mixed lined
recto-concave angles ABD. and AEG. are like
angles. And by the conſtruction it is apparent
that the two recto-concave angles A B D. and
A E D. are equal: and that the two recto-con-
cave angles A E G. and A E D. are unequal:
and that the recto-concave angle A E D. is
greater than the recto-concave angle *A E G.*
and in all like caſes it is always ſo; however
the difference of the two angles muſt neceſſarily
be leſs than any right lined angle, becauſe all
ſuch citradiametral concavo-convexe angles of
contact, as G E D. are always leſs than any
right lined angle; as is conſequent to what hath
been demonſtrated in Geometry, which was to
be ſhewn. Whence we may clearly obſerve
that

that fimilitude of Figures lies chiefly in the pro-
portionality and like pofiture of homologal
fides, in refpect of parallelifme and coincidence,
without impofing any other neceffity for the e-
quality of anfwering angles, then as it may
confift with the proportionating and like pofi-
ting of the homologal fides and lines. And fuch
inequality of the anfwering angles, as is requi-
fite to the proportionating and alike pofiting of
the homologal fides and lines in like and une-
qual mixed, or crooked lined figures, is fo far
from being inconfiftent with their figurative fi-
militude, that they cannot without it, under in-
equality keep fimilitude in their figuration.
And though the inequality, which is between
angle and angle be lefs then that which is, or
may be, between the Homologal fides and lines;
yet the inequality of the angles is more different,
being an inequality without proportion, where-
as the inequality of the homologal fides and
lines is ever according to proportion. Upon
the whole, it is not equality that generally makes
angles to be like, for a right lined, and an Ifo-
clitical concavo Convexe may be equal angles,
but never can be like, nor were ever fufpected
to be fo: but that which makes angles to be like
is rather their being contained under homologal
fides, pofited fo as to conftruct a like & homolo-
gal figuration. And this whole matter depends
upon what I before hinted, *Viz.* the figuration of
lines

lines and angles; *Sc.* rectitude, being one single, simple, figuration of lines incapable of any variety, like angles under right lines are always equal, and never can be unequal: but curvature, being infinitely variable, those crooked lines are said to be like, *i. e.* homologal, whose construction is like, so as in like figures upon a common center, to set homologal sides and lines proportionably equidistant, or coincident; as circumferences of like though unequal circles, ellipses, &c. And so under a thousand inequalities such mixed and crooked lined angles may be like: as in Fig. 21. the recto-concave angles A E G. and A E D. being unequal, are both like to the angle *A* B D. and so is every angle how different soever, if contained under a diameter and a circumference. And indeed the figuration of angles, being incompleat, and the length of their sides undetermined, neither parallelisme, nor coincidency, nor proportionality, nor homologal posture, can, when they are unequal, be conceived in their sides, without special relation to some compleat Figure and its Center: so the recto-concave angles *A* E G. and *A* B D. in Fig. 21. are like, as conceived to be each contained respectively under a diameter and a circumference, and so upon a common center positable into parallelisme, coincidence and proportionableness, and all possible likeness and homo-

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logalnefs of figuration. In right-lined angles,
where homologal and like angles are always e-
qual ; for the fame reafon every angle equal to
a right-lined angle, is not prefently a like an-
gle: but a thoufand equal angles , are all, ever,
and to all purpofes, unlike ; as two equal ifo-
clitical right lined and crooked lined angles, be-
caufe they can never be coapted to be anfwering
angles , in like Figures , or to fet their contain-
ing fides homologally , and in parallelifme, or
coincidence.   That equality of anfwering
angles is not fo of the effence of like Fi-
gures, as proportionality of fides , and anfwer-
ing lines , with their parallelifme , or coinci-
dency : only from the propriety of like plane
Figures, follows an equality in all like right-
lined angles ; and in like curve lined Fi-
gures, that their inequality is ever lefs than the
leaft right-lined angle.   Hence therefore ap-
pears that from the fimilitude which is in une-
qual circles, the equality between angles of femi-
circles & right right-lined angles is not effectu-
ally proved. And notwithftanding any thing in
thofe arguments tendred aud proved , every
recto-concave angle contained under a concave
arch of a circle and a right-line, which is per-
pendicular to the right line tangent of the arch
at the angular point , is greater than any right-
lined acute angle, and lefs than a right right-
lined angle : and the recto-convexe angle con-
tained

tained under the convexe arch and the right
lined tangent is lefs than any right lined angle
whatfoever : and the other recto-convexe angle
contained under the convexe arch and the right
line , which is perpendicular to the right line
tangent at the angular point , is greater than a
right right-lined angle , and lefs than any ob-
tufe right-lined angle whatfoever. And where-
as you object that if, as in Fig. 12. the right line
K G A be the diameter of the circle K A D. and
AB the right line tangent , then K A B is a
right right lined angle : and the recto-convexe
angle of contact D A B is no part of the right
right lined angle K A B. that therefore the an-
gle of the femicitcle K A D is ftill equal to a
right right lined angle , becaufe what is taken
out of it was no part of it.   I anfwer, the recto-
convexe angle of contact D A B is indeed no
proportionable part of the right right-lined an-
gle K A B , but yet it is truly a part , though
improportionable , and fo mathematically he-
terogeneal : for if it had been no part at all, and
nothing , then the angle of the femicircle KAD.
(nothing being taken out of the right right-li-
ned angle K A B, but the recto-convexe angle
of contact D A B , which is faid by you to be
nothing, and no angle) it fhould ftill remain a
right right-lined angle ; which is not by any
afferted , the contrary being fo manifeft : be-
fides that the feparability of the recto-convexe

an-

angle D *A* B from the recto-concave angle *KAD*
makes clear and certain, the truth of its being
a part of the right-lined angle KAB.

And likewise from what hath been before
declared in our opening the nature of a plane
angle may clearly appear, that we are not to
underſtand that a plane angle is meerely the an-
gular point, or meerely in the angular point,
as contradiſtinguiſhed from the containing
ſides, though it there terminate, or thence
have its riſe; but angles are in the habitude
of the concurring, containing and inclined
ſides : *Viz.* the habitude which they hold each
to other all along their tendency unto the an-
gular point, or their riſe from thence, if we
would have the full notion, inclination and
figuration of an angle. For there is often a great
inequality and vaſt imparity between the in-
clination, ſometimes of one part of the con-
taining ſide to the other containing ſide, and
the inclination thereunto, of other parts of the
ſame firſt containing ſide; as may appear in all
mixed lined, mixed crooked-lined and all other
aniſoclitical angles. And the nature of an angle
conſiſting in inclination as well as in concur-
rence, though concurrence may be and is in
a point, and inclination at a point, yet incli-
nation muſt be in the lines and of the lines, and
cannot be in a point ſeparately. And methinks
the nature of an angle, and its inclination, is

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scarcely so fully held forth, when the inclination
of the two lineary sides containing it , as if the
sides were not therein concerned , is ordered to
be observed only in the angular point , and not
out of it: because, as you say, though you urge it
to the contrary , many times out of the angular
point in the containing sides , no two points can
be shewen in the one side, where it hath the same
inclination unto the other. Certainly in an indi-
visible, such as is the angular point, if abstractly
considered , it were vain to expect and impossi-
ble to observe any inclination : and no doubt as
the magnitudes inclined are without the angular
point , so is also the inclination: though as they
terminate in the angular point, so doth the incli-
nation. So the angle of a semi circle is not the
common terme of the diameter and the semi cir-
cumference, excluding the diameter and the semi-
cir-cumference for then in an abstracted point
it should be possible to observe an inclination ;
and a point being indivisible should be inclined
unto its self ; which is not convenient to assert :
but rather the angle of a semi circle is the in-
clination of the semi circumference to the dia-
meter , terminated in the angular point , which
is common to both. And whereas it is said that
out of the angular point, no two points can be
shewen in the diameter, at which the diame-
ter is equally inclined to the semi circumfe-
rence, nor in the semi circumference where it
is

is equally inclined to the diameter ; all is allow-
ed and averred as glosseably true , and this
is that which makes the great difference be-
tween anisoclitical and isoclitical angles , and
renders it so impossible to give an anisoclitical
angle equal to an isoclitical angle . For in iso-
clitical angles the inclination of the sides, the
one unto the other , is at all points the same
without any variation ; as every where appears
by the interjected arches of circles drawn upon
the angular point as Center : but in anisocli-
tical angles , at every several point , the one
containing side hath several and different incli-
nations to the other containing side : which is
the cause that isoclitical angles may possibly
and easily be given , sometimes greater , and
sometimes lesser then anisoclitical angles , but
never equal : because in the one the inclina-
tion of the containing sides still varieth , in the
other not at all. And if the whole nature of an
angle lye in the angular point , without exten-
ding the habitude of it farther into the produ-
ction and figuration of the containing sides ; it
will not only be necessary for us to yield unto
you that recto-convexe angles of contact are
not quantitative ; but besides both you and we ,
contrary to what we have alwayes hitherto
judged , shall be constrained to acknowledge
that there is no quantitativenes . neither in
crooked-lined , nor right-lined , nor any other
angles

angles whatſoever , whether ſuperficial , in
one , or ſeveral plaines , or ſolid. And can any
thing be more horrid then to ſay , the quantity
of angles is not to be meaſured by the divarica-
tion of the ſides at the angular point , but by
their divarication in the angular point , where
they have none at all ? But yet though it is thus
evident , that the inclination of the ſides at the
angular point , may and frequently is much leſs
or greater then the inclination of the ſame ſides
at other points ; which , as is above hinted ,
is not to be left out in the full , genuine and
clear conſideration of the nature of angles , their
kinds , figurations and quantities : however
the inclination of the ſides at the angular point ,
is that which is moſt uſually enquired after ,
and moſt uſeful to be ſearched and obſerved in
Geometry ; for the diſcoveries which are from
thence made of lines , how they fall coinci-
dently , or within , or without others. To the
objection that in fig. 12. the right-lined tangent
AB and the arch AL make all one and the ſame
equal inclination to the right-line ſecant AC
in the common angular point A ; and that
therefore the right-lined angle BAC under
the right-lined ſecant AC and the right-line
tangent AB is equal to the mixed-lined recto-
concave angle EAL under the right-line ſecant
AC and the arch AL ; I anſwer as before , in-
clination is not in the angular point abſtractly
con

considered without regard to the sides passing
out of it, but inclination is the relative situa-
tion which the concurring sides have at the
angular point: at least that is their inclination
there: for a point to a line can have no incli-
nation: it may have distance from the line,
but cannot be inclined unto it, because of its
indivisibility. And having already shewn the
inclination of side to side to be of the essence,
notion and nature of an angle; a little may be
reply enough to all those hypersceptical ob-
jections which are founded upon the imagi-
nation of an angle in a right-line, or any incli-
nation, or angularity imagined between a right
line and a point, especially the point being in
the right-line. And equality, or inequality of
angles is not, nor can be judged of by the so
abstractly considered angular point; in which
a thousand several sides of several and unequal
angles may meet indifferently; but the judgment
of the magnitude and equality and inequality of
angles is from the sides, and the order of the
divarications, in which they passe, especially
first of all, from the angular point. Besides how
strangely is it taken for granted, without pro-
ving, that the right-line tangent AB and the
arch AL are equally in the angular point A in-
clined unto the right-line secant AC? If that
could be once proved, the concern of it would
turn the scales of the controversy: but demon-

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ſtration is ſo clear to the contrary , that as
without proof it is not fit to be admitted , ſo
for the proof of it , I know nothing can be pro-
duced beſides an utter deſpair of ever making
it out.

For if the congruency of the ſides termina-
tively in the angular point , were ſufficient to
conſtitute equality in angles , it appears not how
any angles meeting in the ſame or different
angular points could be unequal : every point
by reaſon of its indiviſibility being incapable of
inequality , as well as inclination.

And if all ſuch angles ſo conſtituted by the
falling within , or without of the ſides , ſhall
be doubted and queſtioned whether they be
true and quantitative angles , and whether the
addition , or ſubduction of them be able to
diverſify other angles and their quantities ; all
the pains of the Geometricians to prove the
intracadency and extracadency of the angular
ſides from the ſame augular point , were vain
and to no purpoſe ; the angles remaining alto-
gether the ſame and equal , whether ſuch an-
gles of contact be added to them , or taken from
them. But yet though the true , full and ge-
nuine nature of an angle conſiſt in the mutual
habitude and inclination of the containing and
concurring ſides , it is not ever neceſſary to con-
ſider it with ſuch a largenes in Geometry. The
inclination which the ſides bear mutually each
to

to other at the angular point is out of doubt that
which is of moſt conſtant neceſſity , higheſt
concern and uſefulnes in all angles to be obſer-
ved. It is true in iſoclitical angles , the incli-
nation of the two containing ſides being every
where the ſame and equal , it is indifferently
by Geometricians taken by a circle whoſe center
is in the angular point , of what diameter
ſoever thereunto applyable , and at what ſame
diſtance ſoever from the angle , or at what ſame
longitude ſoever from thence in the ſides the
points be , at which their inclination is ob-
ſerved by intercepted arches : and the ſame two
points , terminating the intercepted arches at
which their mutual inclination is obſerved ,
conſtantly offer themſelves together , whether
you take points at equal diſtance from the an-
gular point , or intercepting in the ſides equal
longitudes between them and the ſame angular
point. But in aniſoclitical angles ; the inclina-
tion of the aniſoclitical ſides varying ſtill in the
continuity of their production , if we uſe the
former method of meaſuring the inclination of
the ſides by intercepted arches of a circle drawn
upon the angular point as center ; in them
therefore Geometricians concern themſelves
little further then to obſerve the mutual incli-
nation of the ſides at the very point of their
angle : and not at any other points in the ani-
ſoclitical ſides , ſave only the point of their

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concurrence ; becaufe of the couftant varia-
tion of their inclinaticn , both in refpect of
fuch , and every other method and way of
meafuring , according to the continuity of their
production. And by arches of circles drawn
upon the angular point as center , it is impof-
fible to meafure the inclination of the anifocli-
tical lines at the point of their concurrence ; the
only way therefore which remains unto Geome-
tricians to meafure fuch anifoclitical angles, *i. e*,
the inclinaticn of their fides at the point of their
concurrence , is by obferving the lines in what
order they depart from the angular point : *Viz.*
which line falls within , which without , and
which is coincident with that unto which it is
compared. So in fig. 12. if AHF and ADK be
equal circles touching in the point A: and G the
center , and *A*GK the diameter of the circle
ADK:and *AEL* the arch of a greater circle touch
ing both the former circles in the fame point
A and with its concave fide at *A* refpecting the
center G, and with its convexe fide at *A* re-
fpecting the circle *HAF* : alfo if *AB* be a right-
line tangent touching all the three former cir-
cles in the point *A*, and the right-line *ADEC*
cutt the circle *ADK* in the point *D*, and the
circle *AEL* in the point *E :* here the Geome-
trician demonftrates the angle K*AB* to be a
right right lined angle , and that the arch *AF*H
palleth out of the angular point *A* without and
beyond

beyond the right-line tangent _AB_ , and with-
out the right right-lined angle K_AB_: and that
the arch _AEL_ passeth from the angular point
_A_ within, or on this side of the right-line tan-
gent _AB_, and within the right right-lined angle
KAB, but without the arch ADK, and with-
out the angle of the semi circle KAD: and
that the right-line _ADEC_ passeth from the an-
gular point _A_ within the arch _A_DK, and
within the angle of the semi circle K_A_D. _A_nd
this is all which is intended in Geometry when
recto-convexe and such like angles of contact
are said to be less then the least right-lined angle:
not that there is any proportion between any
angles of contact and right-lined angles; for
there is none: but that the sides of any such
angle of contact are coaptable , both of them
within any the least right-lined angle. In like
manner when one angle of contact is said to be
greater or lesser then another; it is not the in-
tendment of the Geometrician to assert any pro-
portionablenes between them , but only to set
down in what order the inclined lines pass ,
each in respect of the rest , from the angular
point, or point of concurrence: and how each
is consequently more, or less inclined in that
respect, to any one of the rest , though by the
whole kind and without proportion , and with-
out a common way of measuring their quanti-
ties in some indefinite quantity, according to
the

the kind of the meafuring , indefinite quantity:
the femblablenes of faying , one angle of con-
tact is greater , or lefs then another being only
in this, that when one fide is coapted , the
other falls within , or without ; or when neither
fides can poffibly be coapted, both fall within,
or both fall without , fo as the one is really
greater , or lefs then the other , not in propor-
tion , but by the whole kind and heteroge-
neally , as a two foot right-line is leffer then
a four foot fquare , and a two foot right-line
cannot be contained in an inch fquare , though
in an inch fquare may be drawn a line ten thou-
fand times longer , only out of rectitude. *And*
to prove fuch an heterogeneal inequality ,
whether of the greater , or of the lefs between
angles, is that which is frequently fufficient for
many Geometrical purpofes.

Only two things now remain of all your ob-
jections and fcrupulous quæries. *Viz.*

Firft , why in fome quantities , between the
homogeneals , to a given quantity you cannot
give another in what proportion you pleafe
greater or lefs , The anfwer is clear out of the
foregoing difcourfe : becaufe fome quantities
have but a limited extenfibility : and other
quantities have but a limited divifibility. *All*
proportions at pleafure with refpect to any given
quantity , are only affigneable in thofe homo-
geneals whofe quantityes have both an unlimi-
ted

ted extenfibility and an unlimited divifibility ;
either of which failing, though they cannot
be , fave in a limited proportion one to another,
and to every given homogeneal , yet they can-
not be in any proportion , at pleafure, to any
given homogeneal.

And ²ly. for your other *Quæry* ; whether this
controverfy about the recto-convexe angle of
contact concern abftract Mathematicks , whofe
glory ufed to be that it was devoid of Contro-
verfy, locking forth and keeping out all un-
peaceful bickerings with the diamond-key of
doubtlefs demonftration ; or whether it concern
concrete Mathematicks , which for its concre-
tion unto matter cannot fo well free its felf from
the intanglements of doubts and difputes? and
if it concern the more abftract part of the Ma-
thematicks ; what was the firft flip that drew
fo unworthy a difparagement upon that moft
noble piece of learning? In the preceding dif-
courfe you may obferve, I have been free and
clear in my judgement concerning this ; that it
is a controverfy of the purer and more noble
part of the Mathematicks : and therefore of
the higher concern , that it be brought to a fair
decifion and irrefragable diremption : and that
the firft fpark from whence all this heat after-
ward arofe, was at firft ftruck out of the du-
bioufnes and æquivocation of the word homo-
geneity , mentioned in certain Mathematical
defi-

definitions and Poftulates; but of it felf no where
expreffely defined in the Mathematicks ; ex-
cept we fhall take one of the Puftulates , for its
definition caft into the form of a Poftulate ; or
as cryptically implying that definition and the
poftulate to be thereupon immediately formed :
efpecially when they compared homogeneity in
a miftaken fenfe , with fome mathematical
conclufions , which they had obferved to be
fairly demonftrated.

. Pardon this my zeal on the behalf of the old
learned Romane profeffour. And if thefe pa-
pers feem long , remember yours were not
fhort ; and the queftion hath long troubled the
world : and in matters that are new , and fcarce
yet well underftood, to inculcate once and
again the fame things , is not only juftify'd as
allowable , but judged expedient and in fuch
cafes neceffary. If you pleafe , let the whole
paffe for a fufe Comment upon this ; that in
recto-convexe angles of contact is the inclina-
tion required for an angle by the definition of
an angle ; and therefore they are angles and
their fides not coincident : and they are angular
parts of acknowledged angles feparable from
the remaining angular parts ; and there is an in-
finite divifibility in them , fo as they muft be
quantitative : and yet it is demonftrated that
they are improportionable to all right-lined
angles ; therefore they are heterogeneal : which
in-

inference is the more neceſſary, when upon
examination we find nothing ſo convenably to
ſet forth unto us the nature of mathematical ho-
mogeneity, as the homometricalnes and the
thence ariſing proportionablenes of the magni-
tudes, and of mathematical heterogeneity as
their heterometricalnes and improportionable-
nes.